ACCU 2021

VIRTUAL EVENT

C++ and Linear Algebra

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#include <C++>

includecpp.org

What to expect

What is linear algebra?

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What to expect

What is linear algebra?

What is a linear algebra library?

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What is a linear algebra library?

Customising the library

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Customising the library

Applications in colour

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Applications in colour

Applications in geometry

What is Linear Algebra?

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What is linear algebra?

- “The branch of mathematics concerning linear equations and linear functions, and their representation through matrices and vector spaces”

What is linear algebra?

- “The branch of mathematics concerning linear equations and linear functions, and their representation through matrices and vector spaces”
- $a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$

What is linear algebra?

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- $a_1x_1 + a_2x_2 + ... + a_nx_n = b$
- Geometry

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What is linear algebra?

- “The branch of mathematics concerning linear equations and linear functions, and their representation through matrices and vector spaces”
- $a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$
- Geometry
- Colour
- Solving simultaneous equations

What is linear algebra?

- Matrix

What is linear algebra?

- Matrix
- $$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

What is linear algebra?

- Matrix-scalar multiplication

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What is linear algebra?

- Matrix-scalar multiplication
- $$b * \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} = \begin{bmatrix} b*a_{11} & b*a_{12} & \dots & b*a_{1n} \\ b*a_{21} & b*a_{22} & \dots & b*a_{2n} \\ \dots & \dots & \dots & \dots \\ b*a_{m1} & b*a_{m2} & \dots & b*a_{mn} \end{bmatrix}$$

What is linear algebra?

- Matrix addition

What is linear algebra?

- Matrix addition
- $$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & \dots & \dots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{bmatrix} = \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \dots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \dots & a_{2n}+b_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \dots & a_{mn}+b_{mn} \end{bmatrix}$$

What is linear algebra?

- Matrix-matrix multiplication

What is linear algebra?

- Matrix-matrix multiplication
- $$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} * \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & \dots & \dots & \dots \\ b_{n1} & b_{n2} & \dots & b_{nm} \end{bmatrix} = \begin{bmatrix} a_1.b_1 & a_1.b_2 & \dots & a_1.b_n \\ a_2.b_1 & a_2.b_2 & \dots & a_2.b_n \\ \dots & \dots & \dots & \dots \\ a_m.b_1 & a_m.b_2 & \dots & a_m.b_n \end{bmatrix}$$

What is linear algebra?

- Matrix-matrix multiplication
- $$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} * \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \dots & \dots & \dots & \dots \\ b_{n1} & b_{n2} & \dots & b_{nm} \end{bmatrix} = \begin{bmatrix} a_1.b_1 & a_1.b_2 & \dots & a_1.b_n \\ a_2.b_1 & a_2.b_2 & \dots & a_2.b_n \\ \dots & \dots & \dots & \dots \\ a_m.b_1 & a_m.b_2 & \dots & a_m.b_n \end{bmatrix}$$
- A * B != B * A

What is linear algebra?

- Square matrix

What is linear algebra?

- Square matrix
- $[a_{11}\ a_{12}\ \dots\ a_{1n}]$
  $[a_{21}\ a_{22}\ \dots\ a_{2n}]$
  $[\ \dots\ \dots\ \dots\ \dots\ ]$
  $[a_{n1}\ a_{n2}\ \dots\ a_{nn}]$

What is linear algebra?

- Identity matrix

What is linear algebra?

- Identity matrix
- ```
  I = [1 0 … 0]
      [0 1 … 0]
      [ …   …  ]
      [0 0 … 1]
  ```

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  ```

What is linear algebra?

- Identity matrix
- ```
  I = [1 0 … 0]
      [0 1 … 0]
      [ …   … ]
      [0 0 … 1]
  ```
- A * I = I * A = A

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  ```

What is linear algebra?

- Determinant of A = |A|

What is linear algebra?

- Determinant of A = |A|
- Inverse of A = A^{-1}
- $A * A^{-1} = A^{-1} * A = I$

What is linear algebra?

- Determinant of A = |A|
- Inverse of A = A^{-1}
- $A * A^{-1} = A^{-1} * A = I$

What is linear algebra?

- operator+()
- operator-()
- operator*()
- operator/()
- ~~operator++(), operator--()~~
- ~~operator>(), operator<()~~

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What is linear algebra?

- Vector

What is linear algebra?

- Vector
- Single row or single column

What is linear algebra?

- Vector
- Single row or single column
- Inner product

What is linear algebra?

- Vector
- Single row or single column
- Inner product
-
```
[a b] * [x] = (a * x) + (b * y)
        [y]
```

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What is linear algebra?

- Vector
- Single row or single column
- Outer product

What is linear algebra?

- Vector
- Single row or single column
- Outer product
- ```
  [a] * [x y] = [a * x  a * y]
  [b]           [b * x  b * y]
  ```
  ```

What is linear algebra?

- Vector
- Single row or single column
- Abstraction problem

What is linear algebra?

- Vector
- Single row or single column
- Abstraction problem
- Naming problem

What is linear algebra?

- ax + by = e
  cx + dy = f

What is linear algebra?

- ```
  ax + by = e
  cx + dy = f
  ```
- ```
  [a b] * [x] = [e]
  [c d]   [y]   [f]
  ```

What is linear algebra?

```
◈ ax + by = e
  cx + dy = f

◈ [a b] * [x] = [e]
  [c d]   [y]   [f]

◈ M * [x] = [e]
      [y]   [f]
```

What is linear algebra?

- ```
  ax + by = e
  cx + dy = f
  ```
- ```
  [a b] * [x] = [e]
  [c d]   [y]   [f]
  ```
- ```
  M * [x] = [e]
      [y]   [f]
  ```
- ```
  [x] = M^-1 * [e]
  [y]          [f]
  ```

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What is linear algebra?

- ```
  2x + 3y =  8
   x - 2y = -3
  ```
  ```

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- ```
  M = [2  3]
      [1 -2]
  ```

What is linear algebra?

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```
2x + 3y =  8
 x - 2y = -3
```
-
```
M = [2  3]
    [1 -2]
```
- $M^{-1} = |M|^{-1} * \text{adjugate}(M)$

What is linear algebra?

- ```
  2x + 3y =  8
   x - 2y = -3
  ```
- ```
  M = [2  3]
      [1 -2]
  ```
- $M^{-1} = |M|^{-1} * \text{adjugate}(M)$
- ```
  |M| = (2 * -2) - (1 * 3)
      = -7
  ```

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  ```

What is linear algebra?

- ```
  2x + 3y =  8
   x - 2y = -3
  ```
- ```
  M = [2  3]
      [1 -2]
  ```
- $M^{-1} = |M|^{-1} * \text{adjugate}(M)$
- ```
  |M| = (2 * -2) - (1 * 3)
      = -7
  ```
- ```
  adjugate(M) = [-2 -3]
                [-1  2]
  ```

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What is linear algebra?

- $2x + 3y = 8$
  $x - 2y = -3$
- $M = [2 \;\; 3]$
  $\quad\;\;\; [1 \;\; -2]$
- $M^{-1} = -7^{-1} * [-2 \;\; -3]$
  $\qquad\qquad\qquad\;\; [-1 \;\; 2]$

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What is linear algebra?

- $$\begin{aligned} 2x + 3y &= 8 \\ x - 2y &= -3 \end{aligned}$$
- $$M = \begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}$$
- $$M^{-1} = -7^{-1} * \begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix}$$
- $$\begin{bmatrix} x \\ y \end{bmatrix} = -7^{-1} * \begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix} * \begin{bmatrix} 8 \\ -3 \end{bmatrix}$$

What is linear algebra?

- 2x + 3y = 8
x - 2y = -3
- M = [2 3]
[1 -2]
- $M^{-1} = -7^{-1}$ * [-2 -3]
[-1 2]
- [x] = -7^{-1} * [-2 -3] * [8]
[y] [-1 2] [-3]
- [x] = [((-2 * 8) + (-3 * -3)) / -7] = [1]
[y] [((-1 * 8) + (2 * -3)) / -7] = [2]

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What is a Linear Algebra Library?

What is a Linear Algebra Library?

- 68000, fixed point

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What is a Linear Algebra Library?

- 68000, fixed point
- 80286, fixed point, C/C++

What is a Linear Algebra Library?

- 68000, fixed point
- 80286, fixed point, C/C++
- 80486, floating point, C++

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What is a Linear Algebra Library?

- 68000, fixed point
- 80286, fixed point, C/C++
- 80486, floating point, C++
- SSE2, Pentium IV

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- 68000, fixed point
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- SSE2, Pentium IV
- AVX, (Sandy bridge)

What is a Linear Algebra Library?

- 68000, fixed point
- 80286, fixed point, C/C++
- 80486, floating point, C++
- SSE2, Pentium IV
- AVX, (Sandy bridge)
- N4860, P1385

What is a Linear Algebra Library?

- Optimisations available through specialisation

What is a Linear Algebra Library?

- Optimisations available through specialisation
- Matrix size

What is a Linear Algebra Library?

- Optimisations available through specialisation
- Matrix size
- Float

What is a Linear Algebra Library?

- Optimisations available through specialisation
- Matrix size
- Float
- SIMD instruction set

What is a Linear Algebra Library?

- Optimisations available through specialisation
- Matrix size
- Float
- SIMD instruction set
- Cache line size

What is a Linear Algebra Library?

- Optimisations available through specialisation
- Matrix size
- Float
- SIMD instruction set
- Cache line size
- Dense

What is a Linear Algebra Library?

- Matrix

What is a Linear Algebra Library?

- Matrix
- Vector

What is a Linear Algebra Library?

- Matrix
- Vector
- Infix operator overloads

What is a Linear Algebra Library?

- Matrix
- Vector
- Infix operator overloads
- M+M, V-V, a*M, V/a…

What is a Linear Algebra Library?

- Matrix
- Vector
- Infix operator overloads
- M+M, V-V, a*M, V/a…
- V*V, V*M, M*V, M*M

What is a Linear Algebra Library?

- operator >>

What is a Linear Algebra Library?

- operator >>
- operator []

What is a Linear Algebra Library?

- operator >>
- operator []
- m(i,j)

What is a Linear Algebra Library?

- operator >>
- operator []
- m(i,j)
- m[i,j]

What is a Linear Algebra Library?

- operator >>
- operator []
- m(i,j)
- m[i,j]
- m[i][j]

What is a Linear Algebra Library?

- operator *

What is a Linear Algebra Library?

- operator *
- 6 x 9

What is a Linear Algebra Library?

- operator *
- 6 x 9
- operator x

What is a Linear Algebra Library?

- operator *
- 6 x 9
- operator x
- operator ^{x}

What is a Linear Algebra Library?

- operator *
- 6 x 9
- operator x
- operator ^{x}
- 6 ^{x} 9

What is a Linear Algebra Library?

- v * w

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What is a Linear Algebra Library?

- Hadamard product
- `(3, 2) * (4, 2) = (12, 4)`
-
```
[3 2] * [4 1] = [12 2]
[4 2]   [2 2]   [ 8 4]
```

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What is a Linear Algebra Library?

- BLAS (Basic Linear Algebra Subprograms)

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- BLAS (Basic Linear Algebra Subprograms)
- BLAS++

What is a Linear Algebra Library?

- BLAS (Basic Linear Algebra Subprograms)
- BLAS++
-
```
void blas::axpy(int64_t n,        float alpha,
                float const* x, int64_t incx,
                float* y,        int64_t incy);
```

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What is a Linear Algebra Library?

- BLAS (Basic Linear Algebra Subprograms)
- BLAS++
-
```
void blas::axpy(int64_t n,       float alpha,
                float const* x, int64_t incx,
                float* y,       int64_t incy);
```
- Boost.uBLAS

What is a Linear Algebra Library?

asum	vector 1 norm (sum)
axpy	add vectors
copy	copy vector
dot	dot product
dotu	dot product, unconjugated
iamax	max element
nrm2	vector 2 norm
rot	apply Givens plane rotation
rotg	generate Givens plane rotation
rotm	apply modified Givens plane rotation
rotmg	generate modified Givens plane rotation
scal	scale vector
swap	swap vectors

What is a Linear Algebra Library?

- asum
  axpy
  copy
  dot
  dotu
  iamax
  nrm2
  rot
  rotg
  rotm
  rotmg
  scal
  swap

gemv	general matrix-vector multiply
ger	general matrix rank 1 update
hemv	hermitian matrix-vector multiply
her	hermitian rank 1 update
her2	hermitian rank 2 update
symv	symmetric matrix-vector multiply
syr	symmetric rank 1 update
syr2	symmetric rank 2 update
trmv	triangular matrix-vector multiply
trsv	triangular matrix-vector solve

What is a Linear Algebra Library?

asum	gemv	**gemm**	general matrix multiply: C = AB + C
axpy	ger	**hemm**	hermitian matrix multiply
copy	hemv	**herk**	hermitian rank k update
dot	her	**her2k**	hermitian rank 2k update
dotu	her2	**symm**	symmetric matrix multiply
iamax	symv	**syrk**	symmetric rank k update
nrm2	syr	**syr2k**	symmetric rank 2k update
rot	syr2	**trmm**	triangular matrix multiply
rotg	trmv	**trsm**	triangular solve matrix
rotm	trsv		
rotmg			
scal			
swap			

What is a Linear Algebra Library?

asum	gemv	**gemm**	general matrix multiply: C = AB + C
axpy	ger	**hemm**	hermitian matrix multiply
copy	hemv	**herk**	hermitian rank k update
dot	her	**her2k**	hermitian rank 2k update
dotu	her2	**symm**	symmetric matrix multiply
iamax	symv	**syrk**	symmetric rank k update
nrm2	syr	**syr2k**	symmetric rank 2k update
rot	syr2	**trmm**	triangular matrix multiply
rotg	trmv	**trsm**	triangular solve matrix
rotm	trsv		
rotmg			
scal			
swap			

P1673R2: A free function linear algebra interface based on the BLAS

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What is a Linear Algebra Library?

- Eigen

What is a Linear Algebra Library?

- Eigen
- Matrix and vector templates

What is a Linear Algebra Library?

- Eigen
- Matrix and vector templates
- Dynamic or static dimensions

What is a Linear Algebra Library?

- Eigen
- Matrix and vector templates
- Dynamic or static dimensions
- Span

What is a Linear Algebra Library?

- Eigen
- Matrix and vector templates
- Dynamic or static dimensions
- Span
- Member function API

What is a Linear Algebra Library?

```
#include <iostream>
#include <Eigen/Dense>
using namespace Eigen;
using namespace std;

int main() {
  MatrixXd m = MatrixXd::Random(3,3);
  m = (m + MatrixXd::Constant(3,3,1.2)) * 50;
  cout << "m =" << endl << m << endl;
  VectorXd v(3);
  v << 1, 2, 3;
  cout << "m * v =" << endl << m * v << endl;
}
```

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What is a Linear Algebra Library?

- Dlib

What is a Linear Algebra Library?

- Dlib
- Expression templates

What is a Linear Algebra Library?

- Blaze

What is a Linear Algebra Library?

```
#include <iostream>
#include <blaze/Math.h>
using blaze::StaticVector;
using blaze::DynamicVector

int main() {
   StaticVector<int,3UL> a{ 4, -2, 5 }
   DynamicVector<int> b( 3UL );
   b[0] = 2;
   b[1] = 5;
   b[2] = -3;
   DynamicVector<int> c = a + b;
   std::cout << "c =\n" << c << "\n";
}
```

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What is a Linear Algebra Library?

- https://wg21.link/P1385
- Syntax proposal
- Reserve some identifiers
- Boost.QVM

Customising the library

Customising the library

- Element type

Customising the library

- Element type
- Element arrangement

Customising the library

- Element type
- Element arrangement
- `std::math::matrix<float, 3, 3> m1;`

Customising the library

- Element type
- Element arrangement
- `std::math::matrix<float, 3, 3> m1;`
- `std::math::matrix<float> m2;`

Customising the library

- Designing storage engines

Customising the library

- Designing storage engines
- `automatic_storage<T, R, C>`

Customising the library

- Designing storage engines
- `automatic_storage<T, R, C>`
- `dynamic_storage<T, A>`

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Customising the library

- Designing storage engines
- automatic_storage<T, R, C>
- dynamic_storage<T, A>
- std::math::matrix<automatic_storage<float, 3, 3>> m1;

Customising the library

- Designing storage engines
- automatic_storage<T, R, C>
- dynamic_storage<T, A>
- std::math::matrix<automatic_storage<float, 3, 3>> m1;
- std::math::matrix<dynamic_storage<float, std::allocator>> m2;

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Customising the library

- Designing storage engines
- `automatic_storage<T, R, C>`
- `dynamic_storage<T, A>`
- `std::math::matrix<automatic_storage<float, 3, 3>> m1;`
- `std::math::matrix<dynamic_storage<float, std::allocator>> m2;`
-
```
using geometry = automatic_storage<float, 3, 3>;
std::math::matrix<geometry> m1;
```

Customising the library

- mdspan : P0009

Customising the library

- mdspan : P0009
- Multidimensional arrays are a foundational data structure for science and engineering codes, as demonstrated by their extensive use in Fortran for five decades. A multidimensional array is a view to a memory extent through a layout mapping from a multi-index space (domain) to that extent (range).

Customising the library

- mdspan : P0009
- Traditional layout mappings have been specified as part of the language. For example, Fortran specifies column major layout and C specifies row major layout. Such a language-imposed specification requires significant code refactoring to change an array's layout and requires significant code complexity to implement non-traditional layouts such as tiling in modern linear algebra or structured grid application domains.

Customising the library

- mdspan : P0009
- A multidimensional array view abstraction with polymorphic layout is required to enable changing array layout without extensive code refactoring and maintenance of functionally redundant code. Layout polymorphism is a critical capability; however, it is not the only beneficial form of polymorphism.

Customising the library

- mdspan : P0009
- `template <ptrdiff_t… Extents> class extents;`

Customising the library

- mdspan : P0009
- `template <ptrdiff_t… Extents> class extents;`
- `dynamic_extent`

Customising the library

- `matrix_storage_engine<T, extents<R, C>, A>;`

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Customising the library

```
matrix_storage_engine<T, extents<R, C>, A>;
matrix<matrix_storage_engine<float, extents<3, 3>, void>>;
```

Customising the library

- `matrix_storage_engine<T, extents<R, C>, A>;`
- `matrix<matrix_storage_engine<float, extents<3, 3>, void>>;`
- `matrix<matrix_storage_engine<float, dynamic_extents, std::allocator<T>>>;`

Customising the library

- `matrix_storage_engine<T, extents<R, C>, A>;`
- `matrix<matrix_storage_engine<float, extents<3, 3>, void>>;`
- `matrix<matrix_storage_engine<float, dynamic_extents, std::allocator<T>>>;`
- `matrix_storage_engine<T, extents<R, C>, A, L>;`

Customising the library

- `matrix_storage_engine<T, extents<R, C>, A>;`
- `matrix<matrix_storage_engine<float, extents<3, 3>, void>>;`
- `matrix<matrix_storage_engine<float, dynamic_extents, std::allocator<T>>>;`
- `matrix_storage_engine<T, extents<R, C>, A, L>;`
- ```
  matrix<matrix_storage_engine<float, extents<3, 3>, void,
                                          matrix_layout::row_major>>;
  ```
  ```

Customising the library

- `matrix_storage_engine<T, extents<R, C>, A>;`
- `matrix<matrix_storage_engine<float, extents<3, 3>, void>>;`
- `matrix<matrix_storage_engine<float, dynamic_extents, std::allocator<T>>>;`
- `matrix_storage_engine<T, extents<R, C>, A, L>;`
-
```
matrix<matrix_storage_engine<float, extents<3, 3>, void,
                                                matrix_layout::row_major>>;
```
-
```
matrix<matrix_storage_engine<float, dynamic_extents, std::allocator<T>,
                                                matrix_layout::column_major>;
```

Customising the library

- ```
  #include <iostream>

  int main()
  {
      std::cout << 1 + 2.5;
  }
  ```

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  ```

Customising the library

- ```
  #include <iostream>

  int main()
  {
      std::cout << 1 + 2.5;
  }
  ```
- 3.5

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  ```

Customising the library

- ```
  #include <iostream>

  int main()
  {
      std::cout << 1 + 2.5;
  }
  ```
- `3.5`
- `double operator+(int, double)?`
  ```

Customising the library

- ```
  #include <iostream>

  int main()
  {
      std::cout << 1 + 2.5;
  }
  ```
- `3.5`
- `double operator+(int, double)?`
- `double operator+(double, double)`
  ```

Customising the library

- ```
  #include <iostream>
  #include <complex>

  int main()
  {
      std::cout << std::complex<double>(3., 3.);
  }
  ```
  ```

Customising the library

- ```
  #include <iostream>
  #include <complex>

  int main()
  {
      std::cout << std::complex<double>(3., 3.);
  }
  ```
- (3,3)
  ```

Customising the library

```
#include <iostream>
#include <complex>

int main()
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int main()
{
    std::cout << std::complex<int>(3.7, 3.2);
}
```

Customising the library

- ```
  #include <iostream>
  #include <complex>

  int main()
  {
      std::cout << std::complex<int>(3.7, 3.2);
  }
  ```
- (3,3)
  ```

Customising the library

```
#include <iostream>
#include <complex>

int main()
{
    std::cout << std::complex<int>(3.7, 3.2)
                 + std::complex<int>(4, 4);
}
```

Customising the library

- ```
  #include <iostream>
  #include <complex>

  int main()
  {
      std::cout << std::complex<int>(3.7, 3.2)
                + std::complex<int>(4, 4);
  }
  ```
- `(7,7)`
  ```

Customising the library

```
#include <iostream>
#include <complex>

int main()
{
    std::cout << std::complex<float>(3.7, 3.2)
              + std::complex<float>(4, 4);
}
```

Customising the library

- ```
  #include <iostream>
  #include <complex>

  int main()
  {
      std::cout << std::complex<float>(3.7, 3.2)
                      + std::complex<float>(4, 4);
  }
  ```
- `(7.7,7.2)`
  ```

Customising the library

```
#include <iostream>
#include <complex>

int main()
{
    std::cout << std::complex<float>(3.7, 4)
              + std::complex<float>(4, 3.2);
}
```

@hatcat01

Customising the library

- ```
  #include <iostream>
  #include <complex>

  int main()
  {
      std::cout << std::complex<float>(3.7, 4)
                  + std::complex<float>(4, 3.2);
  }
  ```
- (7.7,7.2)

@hatcat01
  ```

Customising the library

```
#include <iostream>
#include <complex>

int main()
{
    std::cout << std::complex<float>(3.7, 4)
              + std::complex<double>(4, 3.2);
}
```

Customising the library

- `binary '+': 'std::complex<float>' does not define this operator or a conversion to a type acceptable to the predefined operator`

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Customising the library

- ```
  using double_33_a = matrix_storage_engine<double, extents<3, 3>,
                      void, matrix_layout::row_major>;
  ```
  ```

Customising the library

```
using double_33_a = matrix_storage_engine<double, extents<3, 3>,
                    void, matrix_layout::row_major>;

using float_33_d = matrix_storage_engine<float, extents<3, 3>,
                    std::allocator<T>, matrix_layout::column_major>;
```

Customising the library

```
using double_33_a = matrix_storage_engine<double, extents<3, 3>,
                      void, matrix_layout::row_major>;

using float_33_d = matrix_storage_engine<float, extents<3, 3>,
                      std::allocator<T>, matrix_layout::column_major>;

matrix<double_33_a> m1 = get_auto_mat();
```

@hatcat01

Customising the library

```
using double_33_a = matrix_storage_engine<double, extents<3, 3>,
                     void, matrix_layout::row_major>;

using float_33_d = matrix_storage_engine<float, extents<3, 3>,
                     std::allocator<T>, matrix_layout::column_major>;

matrix<double_33_a> m1 = get_auto_mat();

matrix<float_33_d> m2 = get_dyna_mat();
```

Customising the library

- ```
  using double_33_a = matrix_storage_engine<double, extents<3, 3>,
                      void, matrix_layout::row_major>;

  using float_33_d = matrix_storage_engine<float, extents<3, 3>,
                     std::allocator<T>, matrix_layout::column_major>;

  matrix<double_33_a> m1 = get_auto_mat();

  matrix<float_33_d> m2 = get_dyna_mat();

  auto m3 = m1 + m2;
  ```
  ```

Customising the library

- matrix_storage_engine<double, extents<3, 3>, void, row_major>;
- matrix_storage_engine<float, extents<3, 3>, void, row_major>;
- => matrix_storage_engine<double, extents<3, 3>, void, row_major>;

Customising the library

- `matrix_storage_engine<double, extents<3, 3>, void, row_major>;`
- `matrix_storage_engine<float, extents<3, 3>, void, row_major>;`
- `=> matrix_storage_engine<double, extents<3, 3>, void, row_major>;`

Customising the library

- matrix_storage_engine<double, extents<3, 3>, void, row_major>;
- matrix_storage_engine<float, extents<3, 3>, void, row_major>;
- => matrix_storage_engine<double, extents<3, 3>, void, row_major>;

Customising the library

- ```
  struct matrix_operation_traits {
    template <typename OTR, class T1, class T2> addition_element_traits;
    template <typename OTR, class T1, class T2> addition_engine_traits;
    template <typename OTR, class T1, class T2> addition_arithmetic_traits;
    template <typename OTR, class T1, class T2> subtraction_element_traits;
    …
    template <typename OTR, class T1, class T2> multiplication_element_traits;
    …
    template <typename OTR, class T1, class T2> addition_element_traits;
  };
  ```
  ```

Customising the library

- ```
  template<class T, ptrdiff_t R, ptrdiff_t C, class COT = void>
  using fixed_size_matrix =
    basic_matrix<matrix_storage_engine<T, extents<R, C>,
      void, matrix_layout::row_major>, COT>;
  ```
- ```
  template<class COT = void>
  using matrix_33f =
    basic_matrix<matrix_storage_engine<float, extents<3, 3>,
      void, matrix_layout::row_major>, COT>;
  ```

Customising the library

- Multiplication

Customising the library

- Multiplication
- $O(n^3)$

Customising the library

- Multiplication
- $O(n^3)$
- Strassen's algorithm – $O(n^{2.807})$

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Customising the library

- Multiplication
- $O(n^3)$
- Strassen's algorithm – $O(n^{2.807})$
- Best result – $O(n^{2.3728639})$

Customising the library

```
struct custom_operation_traits {
  template <typename OTR, class T1, class T2>
  using addition_element_traits =
    std::matrix_operation_traits::addition_element_traits<OTR, T1, T2>;
  template <typename OTR, class T1, class T2>
  using addition_engine_traits =
    std::matrix_operation_traits::addition_engine_traits<OTR, T1, T2>;
  template <typename OTR, class T1, class T2>
  using addition_arithmetic_traits =
    custom_addition_arithmetic_traits<OTR, T1, T2>;
  …
};
```

Customising the library

- `basic_matrix<`

Customising the library

- basic_matrix<
- matrix_storage_engine<

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Customising the library

- `basic_matrix<`
- `            matrix_storage_engine<`
- `                                 element_type,`

Customising the library

- `basic_matrix<`
- `                 matrix_storage_engine<`
- `                                              element_type,`
- `                                              extents<R, C>,`

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Customising the library

```
basic_matrix<
            matrix_storage_engine<
                                 element_type,
                                 extents<R, C>,
                                 allocator,
```

@hatcat01

Customising the library

```
basic_matrix<
            matrix_storage_engine<
                                element_type,
                                extents<R, C>,
                                allocator,
                                layout>,
```

Customising the library

```
basic_matrix<
            matrix_storage_engine<
                                  element_type,
                                  extents<R, C>,
                                  allocator,
                                  layout>,
            matrix_operation_traits>;
```

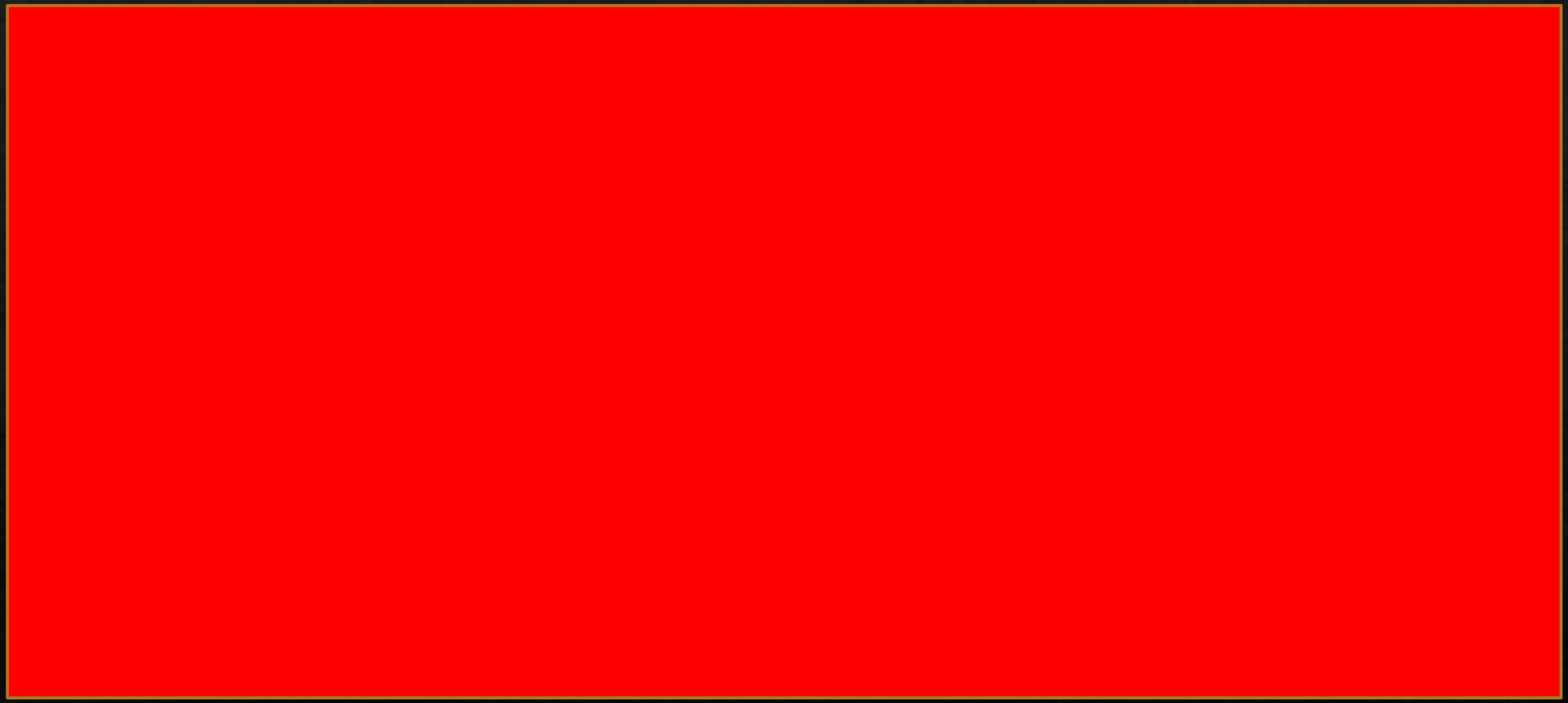
Customising the library

```
template <T> using complex_scalar_storage =
  matrix_storage_engine<std::complex<T>, extents<1, 1>, void>;
template <T> using complex_scalar =
  basic_matrix<complex_scalar_storage<T>>;
complex_scalar<float> c1{2.2f, 3.3f};
complex_scalar<double> c2{4.4, 5.5};
auto c3 = c1 + c2;
```

Applications in colour

@hatcat01

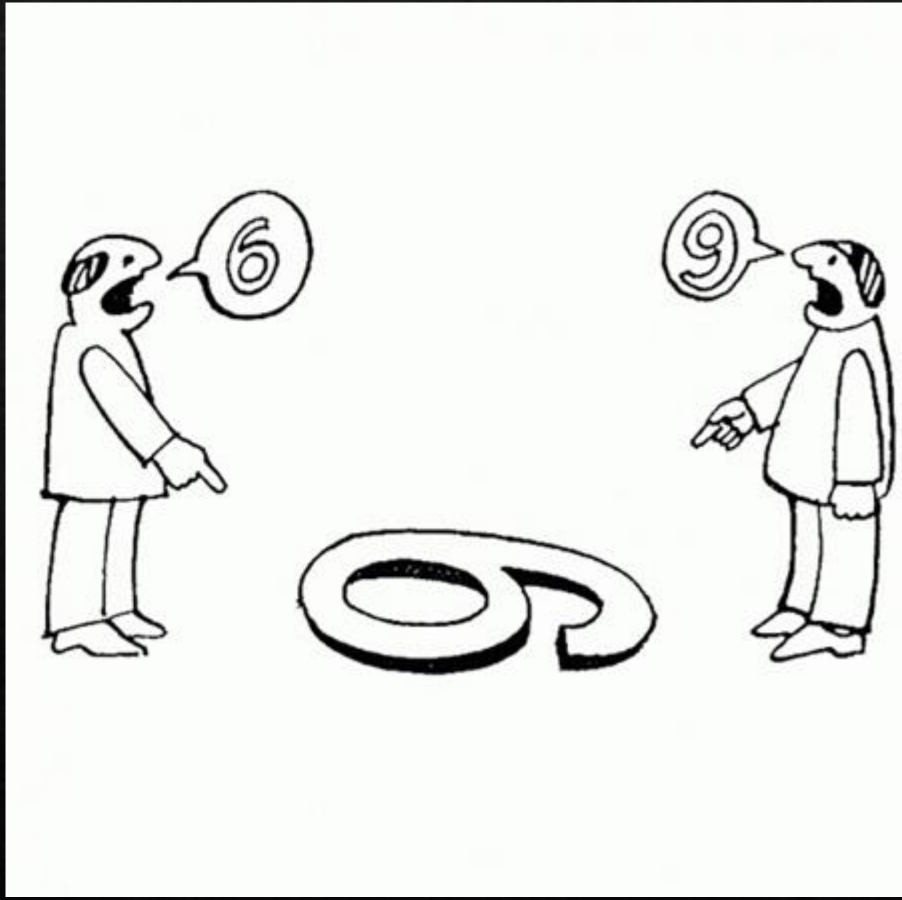
Applications in colour



@hatcat01

Applications in colour

Applications in colour



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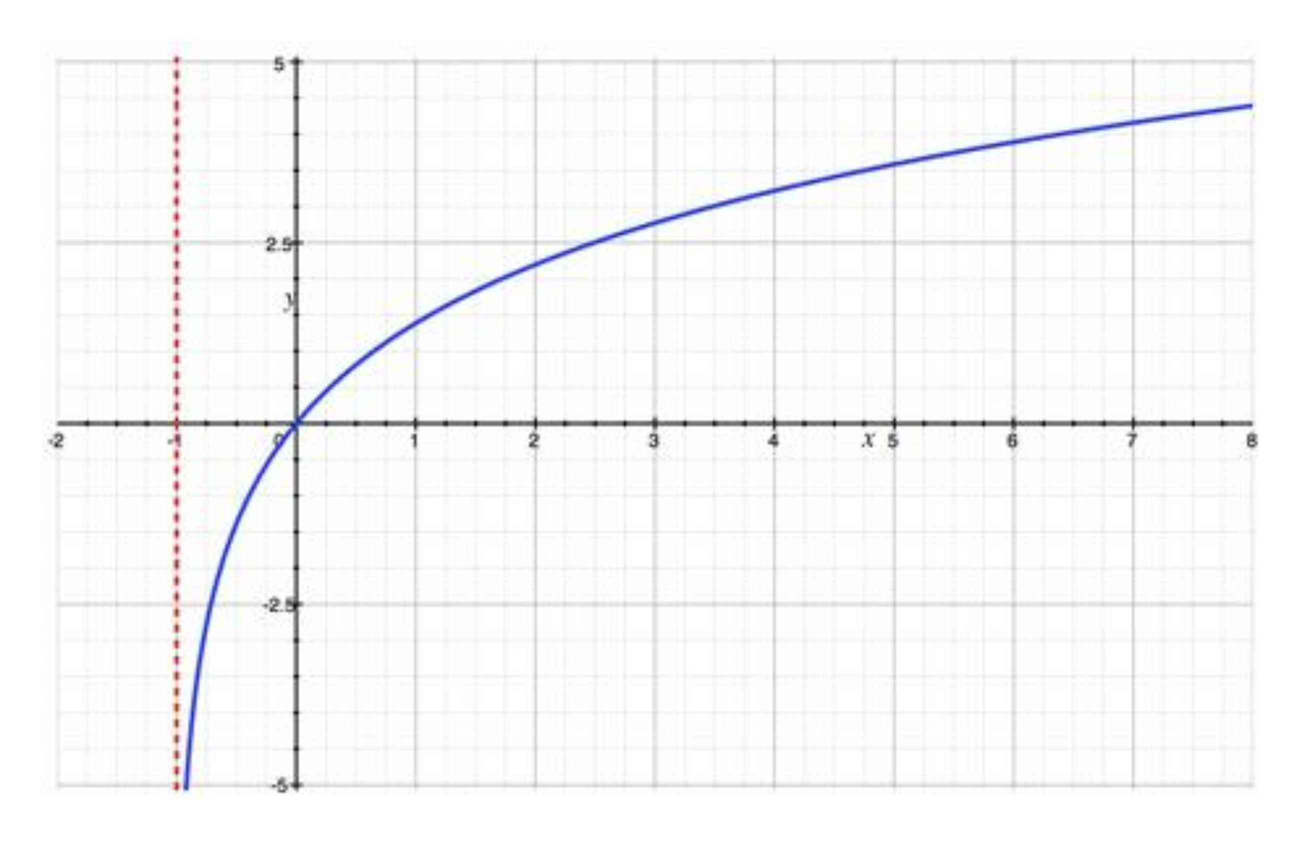
Applications in colour



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Applications in colour

Applications in colour

@hatcat01

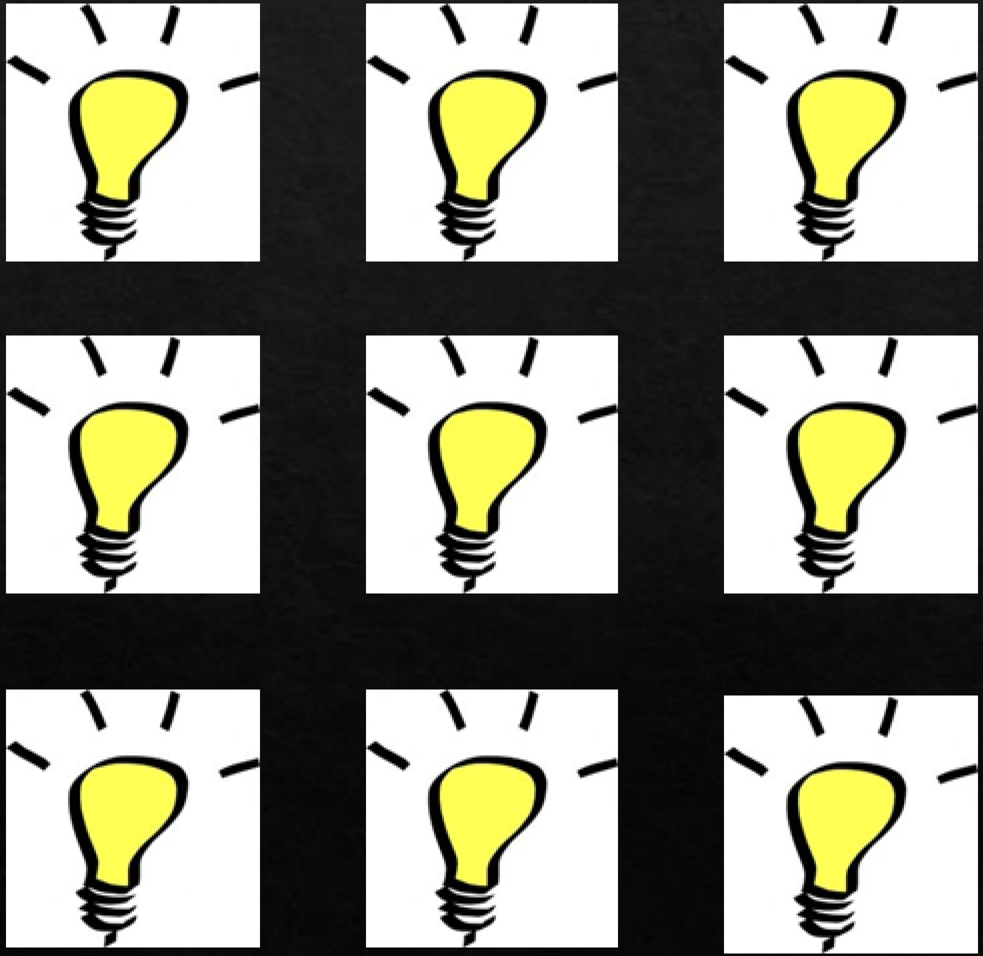
Applications in colour



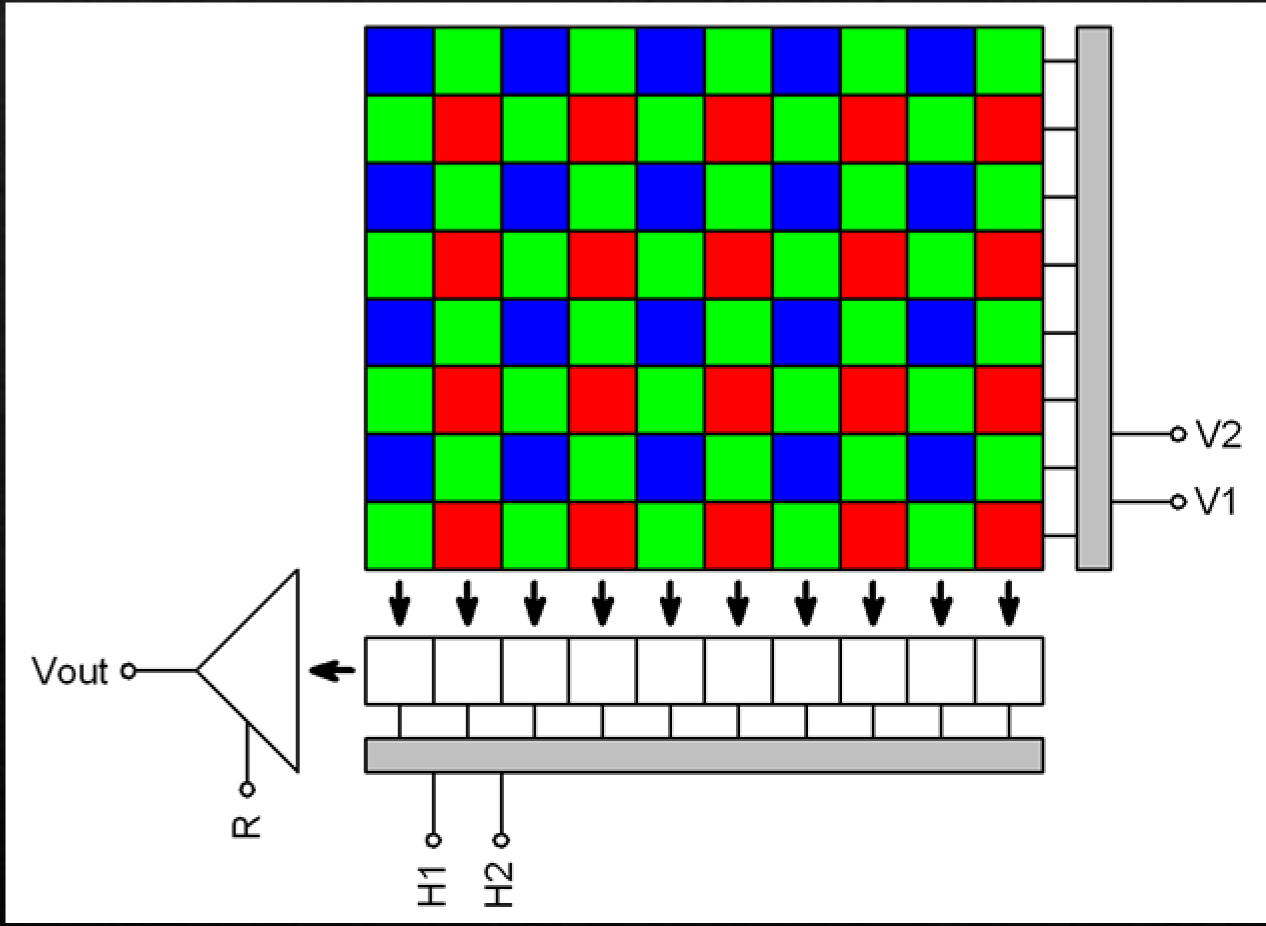





Applications in colour



Applications in colour



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Applications in colour

$\sqrt[2.0]{x}$ $\sqrt[2.0]{x}$ $\sqrt[2.0]{x}$ $\sqrt[2.0]{x}$ $\sqrt[2.0]{x}$ $\sqrt[2.0]{x}$

@hatcat01

Applications in colour

$\sqrt[2.2]{x}$ $\sqrt[2.2]{x}$ $\sqrt[2.2]{x}$ $\sqrt[2.2]{x}$ $\sqrt[2.2]{x}$ $\sqrt[2.2]{x}$

$\sqrt[1.8]{x}$ $\sqrt[1.8]{x}$ $\sqrt[1.8]{x}$ $\sqrt[1.8]{x}$ $\sqrt[1.8]{x}$ $\sqrt[1.8]{x}$

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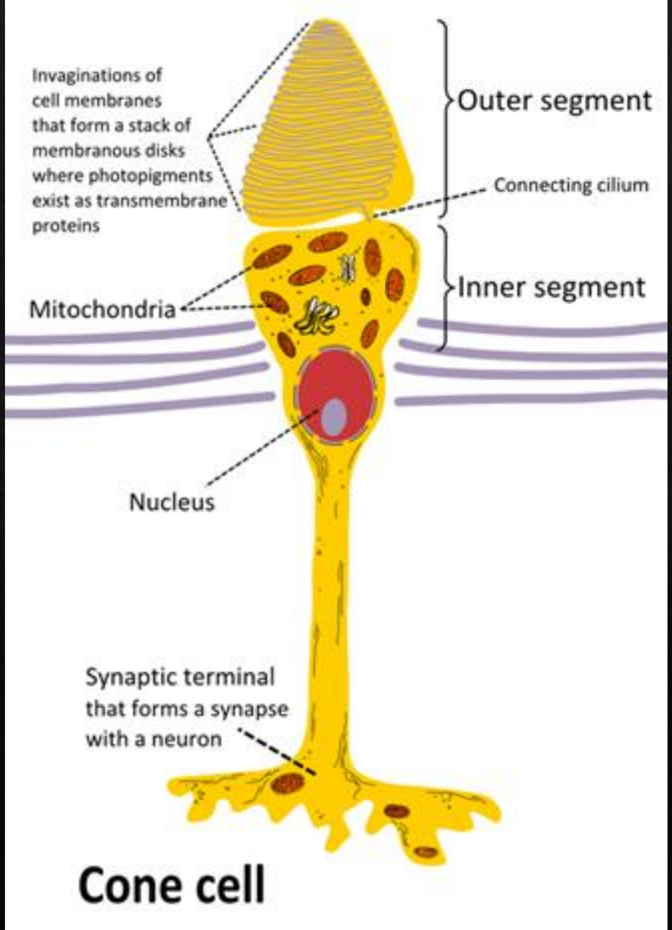
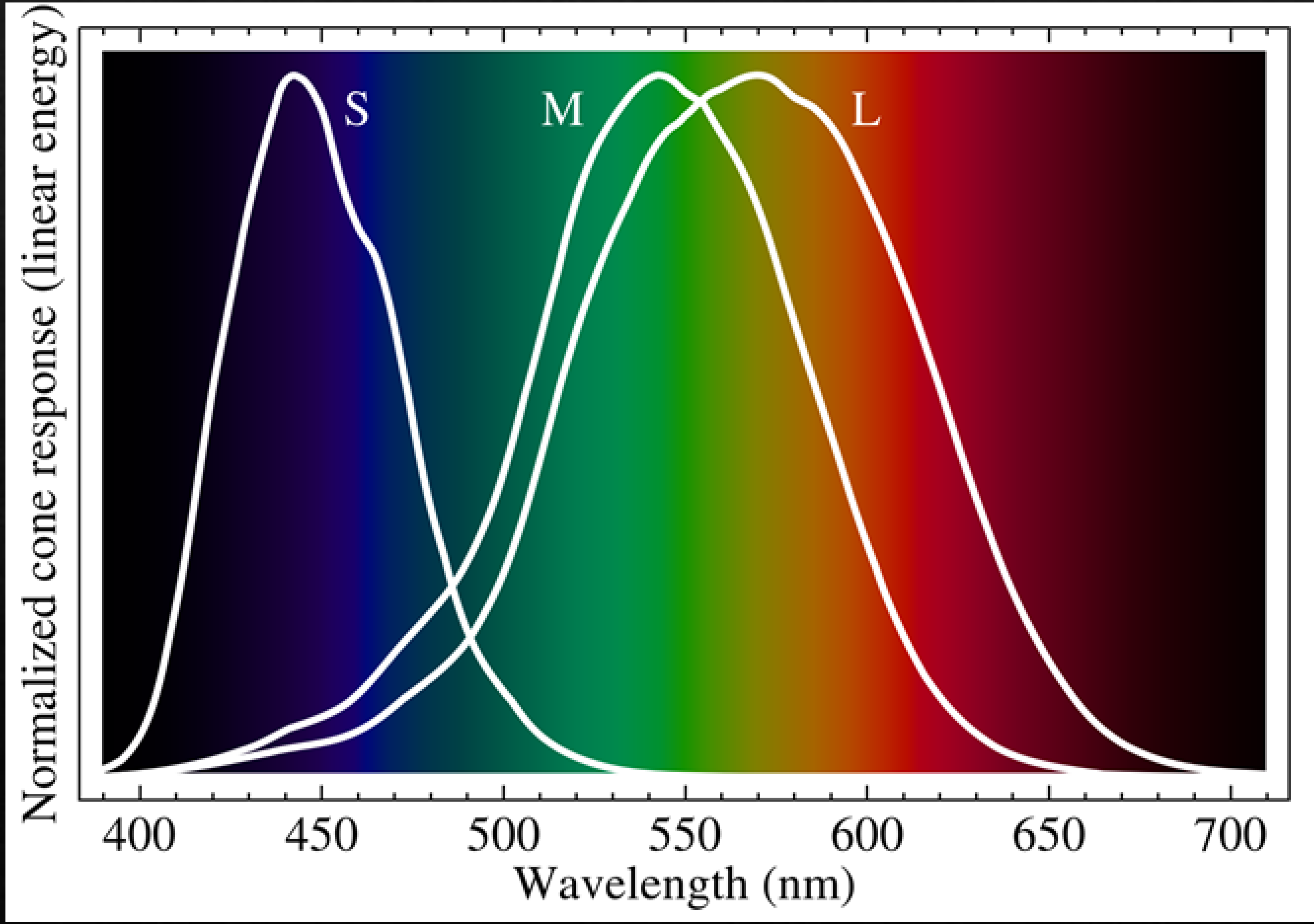
Applications in colour





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Applications in colour

- Take a standard human

Applications in colour

- Take a standard human
- Put them in a standard environment

Applications in colour

- Take a standard human
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- Measure how they perceive electromagnetic waves, via matching the colours of lights

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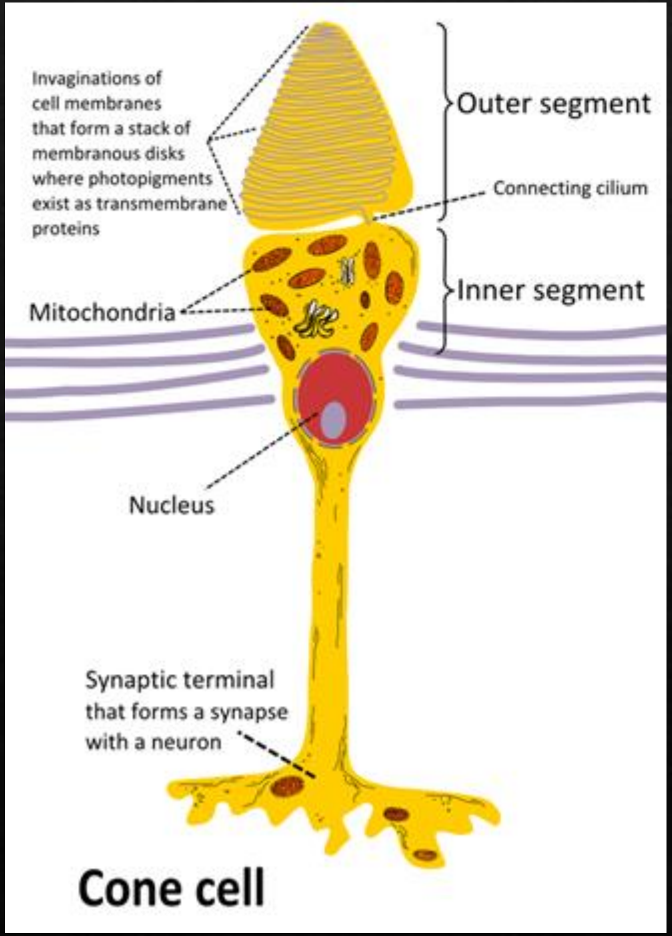
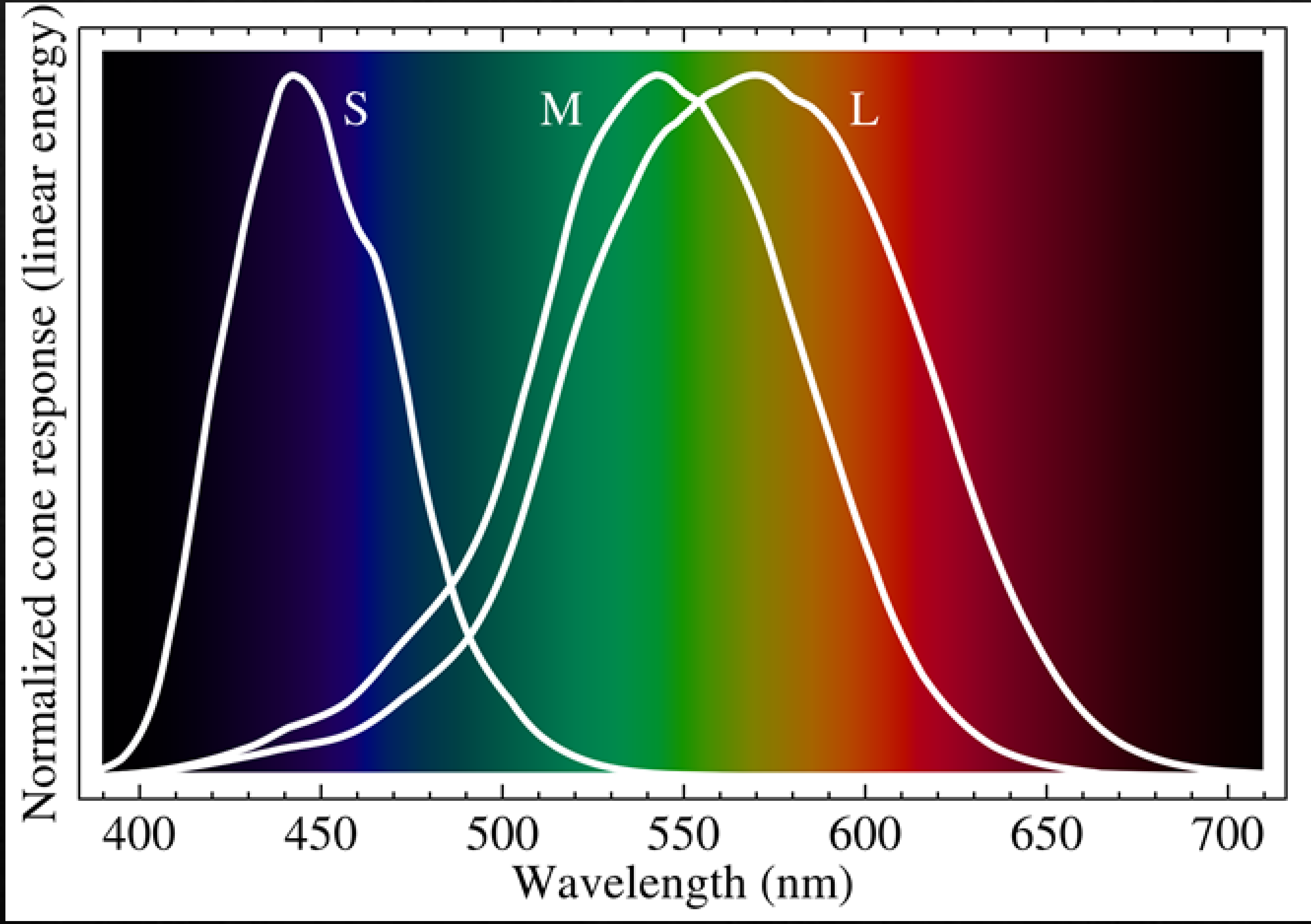
- Build a function that maps wavelengths to perception, giving 3 values (X, Y, Z)

Applications in colour

- Take a standard human
- Put them in a standard environment
- Measure how they perceive electromagnetic waves, via matching the colours of lights
- Build a function that maps wavelengths to perception, giving 3 values (X, Y, Z)
- Add some mathematical constraints (values > 0, Y = relative luminance [0, 100])

Applications in colour





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Applications in colour

- Humans separate colour from brightness

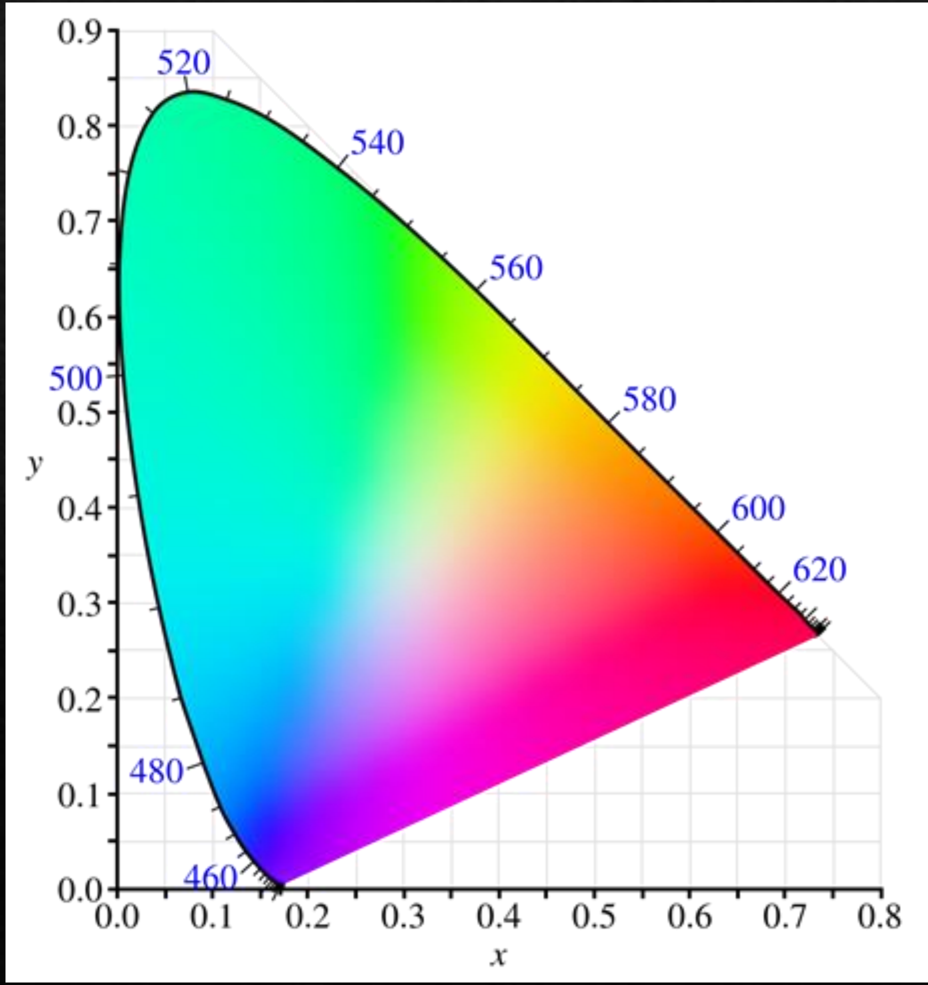
Applications in colour

- Humans separate colour from brightness
- Normalise:
  $x = X / (X + Y + Z)$
  $y = Y / (X + Y + Z)$
  $z = Z / (X + Y + Z) = (1 - x - y)$

Applications in colour

- Humans separate colour from brightness
- Normalise:
  $x = X / (X + Y + Z)$
  $y = Y / (X + Y + Z)$
  $z = Z / (X + Y + Z) = (1 - x - y)$
- xyY colour space
  x and y are colour
  Y is relative luminance

Applications in colour



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Applications in colour

- Small change in a value has the same effect in perceived colour

Applications in colour

- Small change in a value has the same effect in perceived colour
- XYZ values are not perceptually uniform

Applications in colour

- Small change in a value has the same effect in perceived colour
- XYZ values are not perceptually uniform
- Inefficient, like storing sound volume in raw values rather than in dB. 100dB=1^100

Applications in colour

- 1996: Microsoft + HP

Applications in colour

- 1996: Microsoft + HP
- IEC 61966-2-1:1999

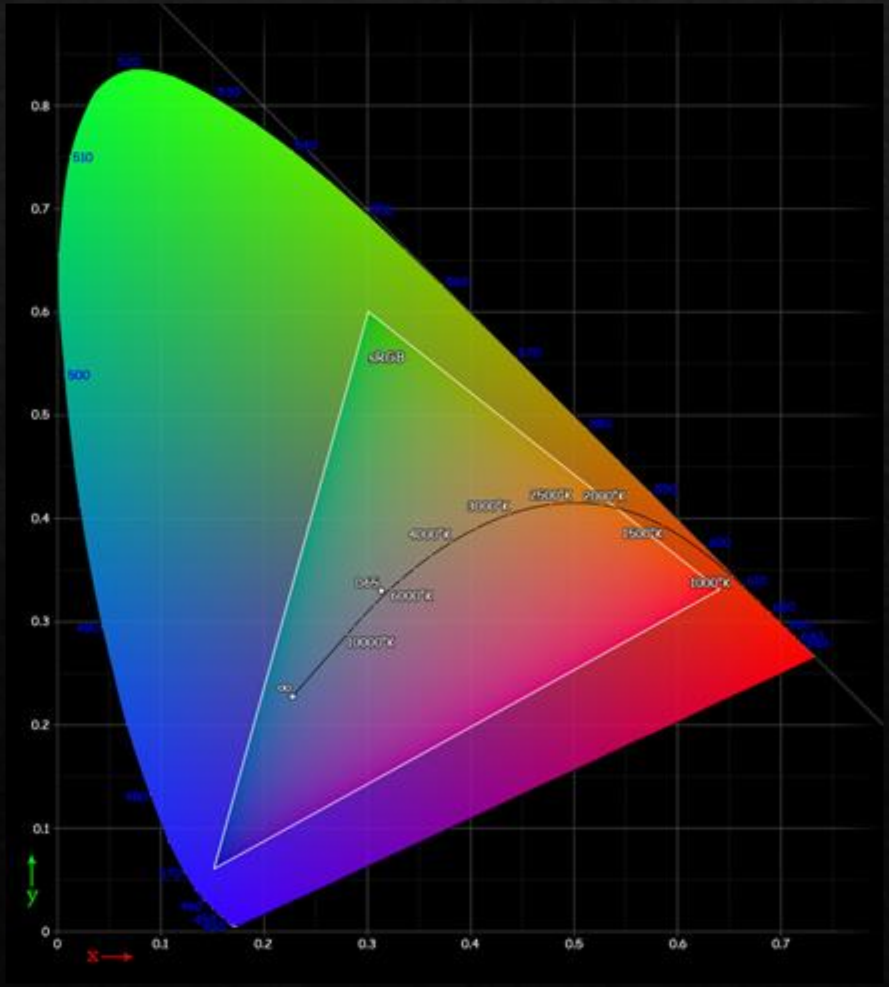
Applications in colour

- 1996: Microsoft + HP
- IEC 61966-2-1:1999
- Default colour space where NO COLOUR SPACE INFORMATION is provided

Applications in colour

Chromaticity	Red	Green	Blue	White point
x	0.6400	0.3000	0.1500	0.3127
y	0.3300	0.6000	0.0600	0.3290
Y	0.2126	0.7152	0.0722	1.0000

Applications in colour



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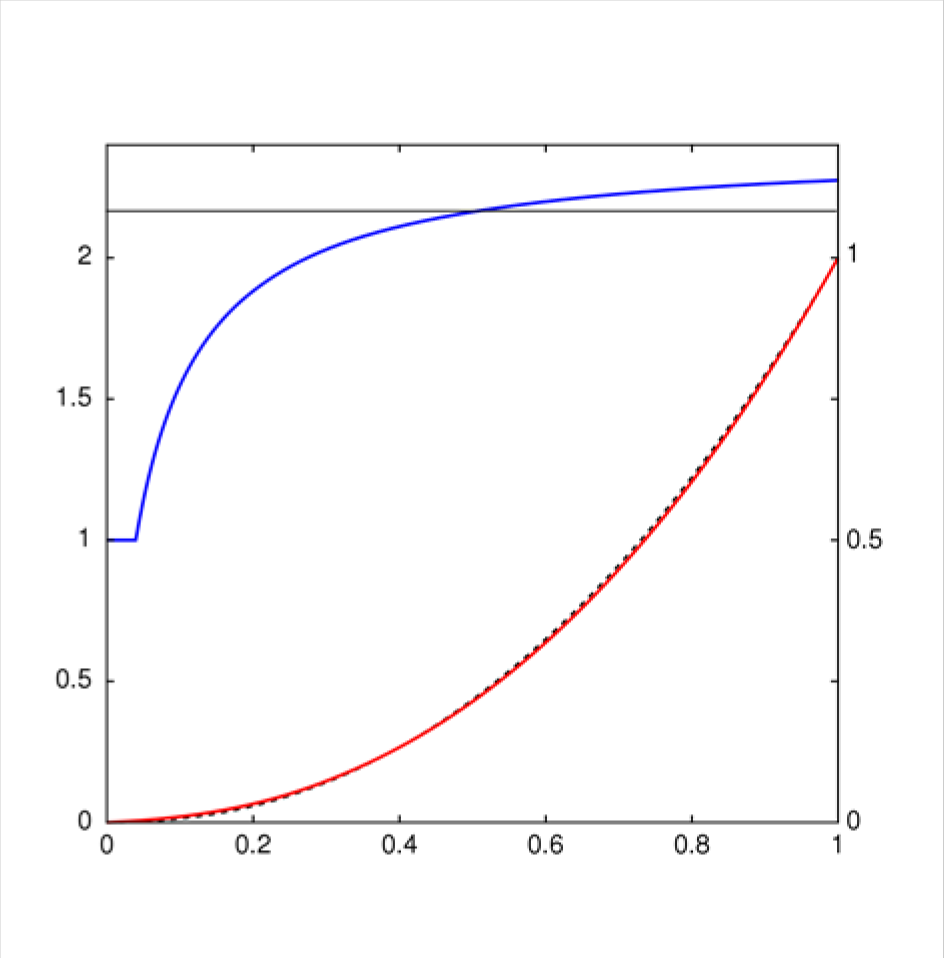
Applications in colour



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Applications in colour

$$\begin{bmatrix} R_{\text{linear}} \\ G_{\text{linear}} \\ B_{\text{linear}} \end{bmatrix} = \begin{bmatrix} +3.24096994 & -1.53738318 & -0.49861076 \\ -0.96924364 & +1.8759675 & +0.04155506 \\ +0.05563008 & -0.20397696 & +1.05697151 \end{bmatrix} \begin{bmatrix} X_{D65} \\ Y_{D65} \\ Z_{D65} \end{bmatrix}$$

$$\gamma(u) = \begin{cases} 12.92u & = \frac{323u}{25} & u \le 0.0031308 \\ 1.055u^{1/2.4} - 0.055 & = \frac{211u^{\frac{5}{12}} - 11}{200} & \text{otherwise} \end{cases}$$

$$\gamma^{-1}(u) = \begin{cases} \frac{u}{12.92} & = \frac{25u}{323} & u \le 0.04045 \\ \left(\frac{u+0.055}{1.055}\right)^{2.4} & = \left(\frac{200u+11}{211}\right)^{\frac{12}{5}} & \text{otherwise} \end{cases}$$

$$\begin{bmatrix} X_{D65} \\ Y_{D65} \\ Z_{D65} \end{bmatrix} = \begin{bmatrix} 0.41239080 & 0.35758434 & 0.18048079 \\ 0.21263901 & 0.71516868 & 0.07219232 \\ 0.01933082 & 0.11919478 & 0.95053215 \end{bmatrix} \begin{bmatrix} R_{\text{linear}} \\ G_{\text{linear}} \\ B_{\text{linear}} \end{bmatrix}$$

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Applications in colour

- Brightness perception is logarithmic

Applications in colour

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- Brightness perception is logarithmic
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- sRGB is defined relative to xyY
- The transfer function is non-linear and expensive

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- Brightness perception is logarithmic
- XYZ defines absolute perceptual colours
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Applications in colour

- Brightness perception is logarithmic
- XYZ defines absolute perceptual colours
- The xyY colourspace is linear
- Linear interpolation is valid on linear colourspaces
- sRGB is defined relative to xyY
- The transfer function is non-linear and expensive
- sRGB is non-linear
- Linear interpolation is invalid on sRGB

Applications in colour

- `(x + y) / 2`

Applications in colour

- `(x + y) / 2`
- `(√x + √y) / 2 < √((x + y) / 2)`

Applications in colour

- $(x + y) / 2$
- $(\sqrt{x} + \sqrt{y}) / 2 < \sqrt{(x + y) / 2}$
- $x = 9, y = 16$

@hatcat01

Applications in colour

- $(x + y) / 2$
- $(\sqrt{x} + \sqrt{y}) / 2 < \sqrt{(x + y) / 2}$
- $x = 9, y = 16$
- $(\sqrt{9} + \sqrt{16}) / 2 = 3.5$

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Applications in colour

- $(x + y) / 2$
- $(\sqrt{x} + \sqrt{y}) / 2 < \sqrt{(x + y) / 2}$
- $x = 9, y = 16$
- $(\sqrt{9} + \sqrt{16}) / 2 = 3.5$
- $\sqrt{(9 + 16) / 2} = 3.535$

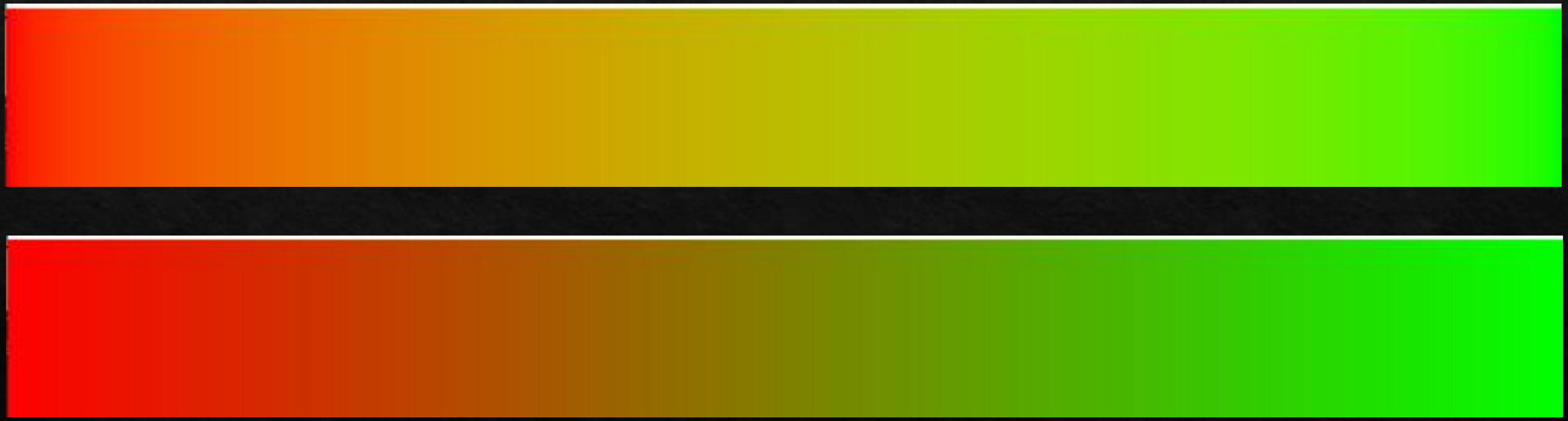
Applications in colour

- `(x + y) / 2`
- `(√x + √y) / 2 < √((x + y) / 2)`
- `x = 9, y = 16`
- `(√9 + √16) / 2 = 3.5`
- `√((9 + 16) / 2) = 3.535`
- ```
  template <class T>
  constexpr std::midpoint(T a, T b) noexcept;
  ```
  ```

Applications in colour

- `(x + y) / 2`
- `(√x + √y) / 2 < √((x + y) / 2)`
- `x = 9, y = 16`
- `(√9 + √16) / 2 = 3.5`
- `√((9 + 16) / 2) = 3.535`
- ```
  template <class T>
  constexpr std::midpoint(T a, T b) noexcept;
  ```
- `constexpr float std::lerp(float a, float b, float t) noexcept`
  ```

Applications in colour



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Applications in colour

```
0 0 0 0 0 0 0 13 13 13 13 13 13 13 13 13 13 13 22 22 22 22 22 22 22 22 28 28 28 28 28 28 34 34 34 34 34 38 38 38 38 42 4
2 42 42 46 46 46 50 50 50 50 53 53 53 56 56 56 59 59 61 61 61 64 64 64 66 66 69 69 71 71 73 73 73 75 75 77 77 79 79 81 8
3 83 85 85 86 86 88 88 90 92 92 93 95 95 96 96 98 99 99 101 102 104 104 105 106 106 108 109 110 112 112 113 114 115 117
117 118 119 120 121 122 124 124 125 126 127 128 129 130 131 132 133 134 135 136 137 138 139 140 141 142 143 144 145 146
147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170 171 172 173 174 175 176
177 178 179 180 181 182 183 184 185 186 187 188 189 190 191 192 193 194 195 196 197 198 199 200 201 202 203 204 205 206
207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223 224 225 226 227 228 229 230 231 232 233 234 235 236
237 238 239 240 241 242 243 244 245 246 247 248 249 250 251 252 253 254 255
Max error 6
Total error 127
```

Applications in colour

- libSDL

Applications in colour

- libSDL
- SFML

Applications in colour

- libSDL
- SFML
- Dear ImGui

Applications in colour

- libSDL
- SFML
- Dear ImGui
- Flash

Applications in colour

- libSDL
- SFML
- Dear ImGui
- Flash
- Unity

Applications in colour

- libSDL
- SFML
- Dear ImGui
- Flash
- Unity
- Godot

Applications in colour

- libSDL
- SFML
- Dear ImGui
- Flash
- Unity
- Godot
- OGRE

Applications in colour

- CRYENGINE

Applications in colour

- CRYENGINE
- MatLab

Applications in colour

- CRYENGINE
- MatLab
- OpenCV

Applications in colour

- CRYENGINE
- MatLab
- OpenCV
- SVG and CSS

Applications in colour

- CRYENGINE
- MatLab
- OpenCV
- SVG and CSS
- Qt

Applications in colour

- CRYENGINE
- MatLab
- OpenCV
- SVG and CSS
- Qt
- Unreal Engine

Applications in colour

```
guy@DESKTOP-69NQDUU:/$ ls
bin  boot  dev  etc  home  init  lib  lib64  media  mnt  opt  proc  root  run  sbin  snap
  srv  sys  tmp  usr  var
guy@DESKTOP-69NQDUU:/$ _
```

Applications in colour

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Applications in geometry

Applications in geometry

- "The branch of mathematics concerned with questions of shape, size, relative position of figures and the properties of space."

@hatcat01

Applications in geometry

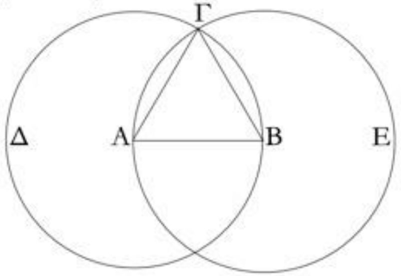
- "The branch of mathematics concerned with questions of shape, size, relative position of figures and the properties of space."

@hatcat01

Applications in geometry

α΄.

Ἐπὶ τῆς δοθείσης εὐθείας πεπερασμένης τρίγωνον ἰσόπλευρον συστήσασθαι.



Ἔστω ἡ δοθεῖσα εὐθεῖα πεπερασμένη ἡ ΑΒ.

Δεῖ δὴ ἐπὶ τῆς ΑΒ εὐθείας τρίγωνον ἰσόπλευρον συστήσασθαι.

Κέντρῳ μὲν τῷ Α διαστήματι δὲ τῷ ΑΒ κύκλος γεγράφθω ὁ ΒΓΔ, καὶ πάλιν κέντρῳ μὲν τῷ Β διαστήματι δὲ τῷ ΒΑ κύκλος γεγράφθω ὁ ΑΓΕ, καὶ ἀπὸ τοῦ Γ σημείου, καθ' ὃ τέμνουσιν ἀλλήλους οἱ κύκλοι, ἐπί τὰ Α, Β σημεῖα ἐπεζεύχθωσαν εὐθεῖαι αἱ ΓΑ, ΓΒ.

Καὶ ἐπεὶ τὸ Α σημεῖον κέντρον ἐστὶ τοῦ ΓΔΒ κύκλου, ἴση ἐστὶν ἡ ΑΓ τῇ ΑΒ· πάλιν, ἐπεὶ τὸ Β σημεῖον κέντρον ἐστὶ τοῦ ΓΑΕ κύκλου, ἴση ἐστὶν ἡ ΒΓ τῇ ΒΑ. ἐδείχθη δὲ καὶ ἡ ΓΑ τῇ ΑΒ ἴση· ἑκατέρα ἄρα τῶν ΓΑ, ΓΒ τῇ ΑΒ ἐστιν ἴση. τὰ δὲ τῷ αὐτῷ ἴσα καὶ ἀλλήλοις ἐστὶν ἴσα· καὶ ἡ ΓΑ ἄρα τῇ ΓΒ ἐστιν ἴση· αἱ τρεῖς ἄρα αἱ ΓΑ, ΑΒ, ΒΓ ἴσαι ἀλλήλαις εἰσίν.

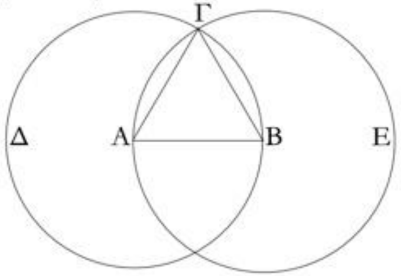
Ἰσόπλευρον ἄρα ἐστὶ τὸ ΑΒΓ τρίγωνον. καὶ συνέσταται ἐπὶ τῆς δοθείσης εὐθείας πεπερασμένης τῆς ΑΒ. ὅπερ ἔδει ποιῆσαι.

@hatcat01

Applications in geometry

α΄.

Ἐπὶ τῆς δοθείσης εὐθείας πεπερασμένης τρίγωνον ἰσόπλευρον συστήσασθαι.



Ἔστω ἡ δοθεῖσα εὐθεῖα πεπερασμένη ἡ ΑΒ.

Δεῖ δὴ ἐπὶ τῆς ΑΒ εὐθείας τρίγωνον ἰσόπλευρον συστήσασθαι.

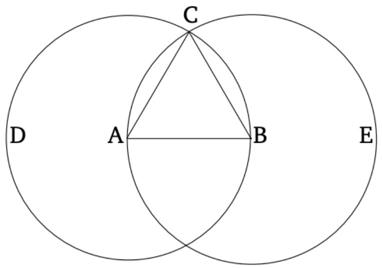
Κέντρῳ μὲν τῷ Α διαστήματι δὲ τῷ ΑΒ κύκλος γεγράφθω ὁ ΒΓΔ, καὶ πάλιν κέντρῳ μὲν τῷ Β διαστήματι δὲ τῷ ΒΑ κύκλος γεγράφθω ὁ ΑΓΕ, καὶ ἀπὸ τοῦ Γ σημείου, καθ' ὃ τέμνουσιν ἀλλήλους οἱ κύκλοι, ἐπί τὰ Α, Β σημεῖα ἐπεζεύχθωσαν εὐθεῖαι αἱ ΓΑ, ΓΒ.

Καὶ ἐπεὶ τὸ Α σημεῖον κέντρον ἐστὶ τοῦ ΓΔΒ κύκλου, ἴση ἐστὶν ἡ ΑΓ τῇ ΑΒ· πάλιν, ἐπεὶ τὸ Β σημεῖον κέντρον ἐστὶ τοῦ ΓΑΕ κύκλου, ἴση ἐστὶν ἡ ΒΓ τῇ ΒΑ. ἐδείχθη δὲ καὶ ἡ ΓΑ τῇ ΑΒ ἴση· ἑκατέρα ἄρα τῶν ΓΑ, ΓΒ τῇ ΑΒ ἐστιν ἴση. τὰ δὲ τῷ αὐτῷ ἴσα καὶ ἀλλήλοις ἐστὶν ἴσα· καὶ ἡ ΓΑ ἄρα τῇ ΓΒ ἐστιν ἴση· αἱ τρεῖς ἄρα αἱ ΓΑ, ΑΒ, ΒΓ ἴσαι ἀλλήλαις εἰσίν.

Ἰσόπλευρον ἄρα ἐστὶ τὸ ΑΒΓ τρίγωνον. καὶ συνέσταται ἐπὶ τῆς δοθείσης εὐθείας πεπερασμένης τῆς ΑΒ. ὅπερ ἔδει ποιῆσαι.

Proposition 1

To construct an equilateral triangle on a given finite straight-line.



Let AB be the given finite straight-line.

So it is required to construct an equilateral triangle on the straight-line AB.

Let the circle BCD with center A and radius AB have been drawn [Post. 3], and again let the circle ACE with center B and radius BA have been drawn [Post. 3]. And let the straight-lines CA and CB have been joined from the point C, where the circles cut one another,† to the points A and B (respectively) [Post. 1].

And since the point A is the center of the circle CDB, AC is equal to AB [Def. 1.15]. Again, since the point B is the center of the circle CAE, BC is equal to BA [Def. 1.15]. But CA was also shown (to be) equal to AB. Thus, CA and CB are each equal to AB. But things equal to the same thing are also equal to one another [C.N. 1]. Thus, CA is also equal to CB. Thus, the three (straight-lines) CA, AB, and BC are equal to one another.

Thus, the triangle ABC is equilateral, and has been constructed on the given finite straight-line AB. (Which is) the very thing it was required to do.

@hatcat01

Applications in geometry

- René Descartes
- b. 31st March 1596
- d. 11th February 1650

@hatcat01

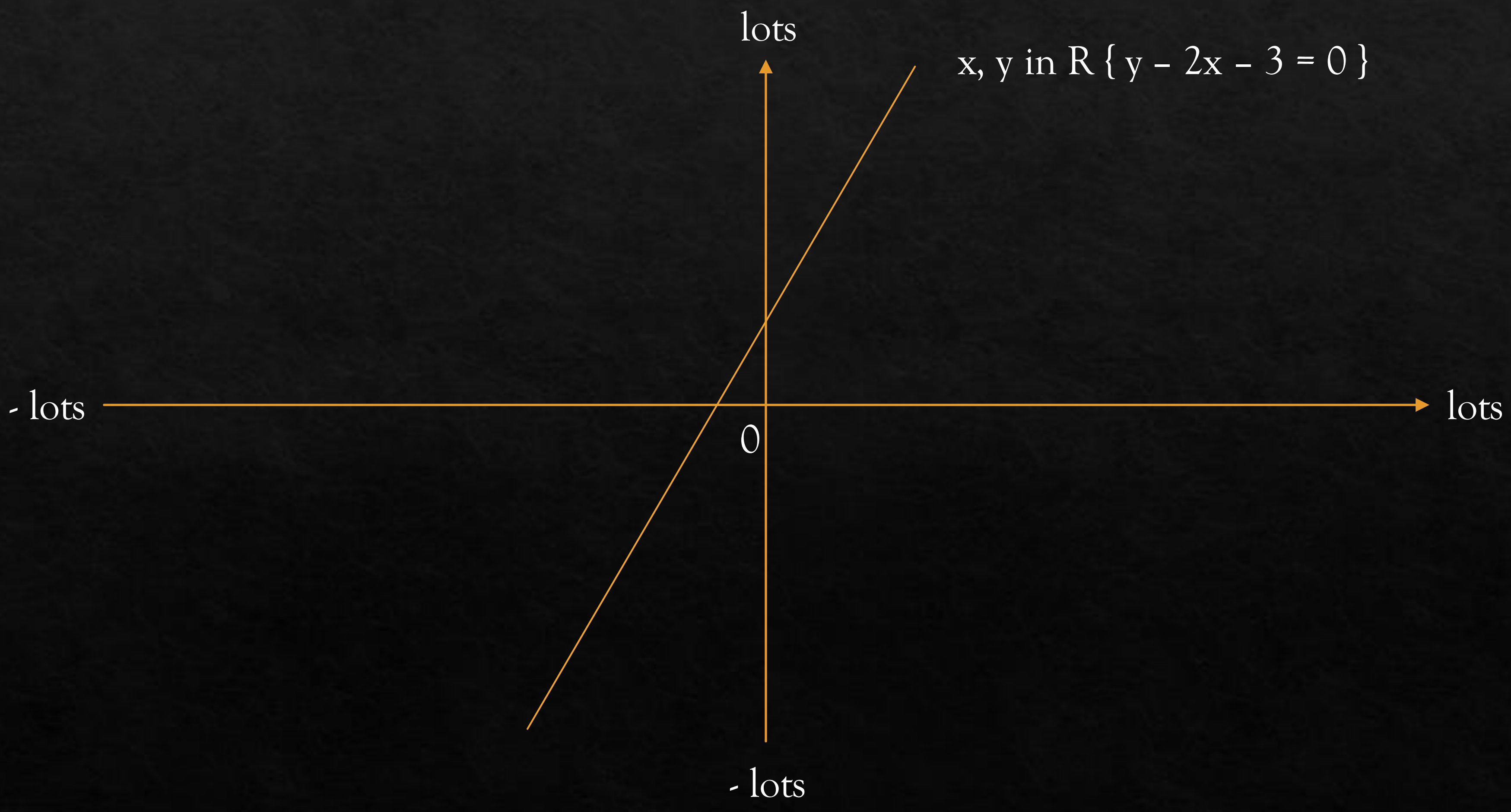
Applications in geometry

@hatcat01

Applications in geometry

@hatcat01

Applications in geometry



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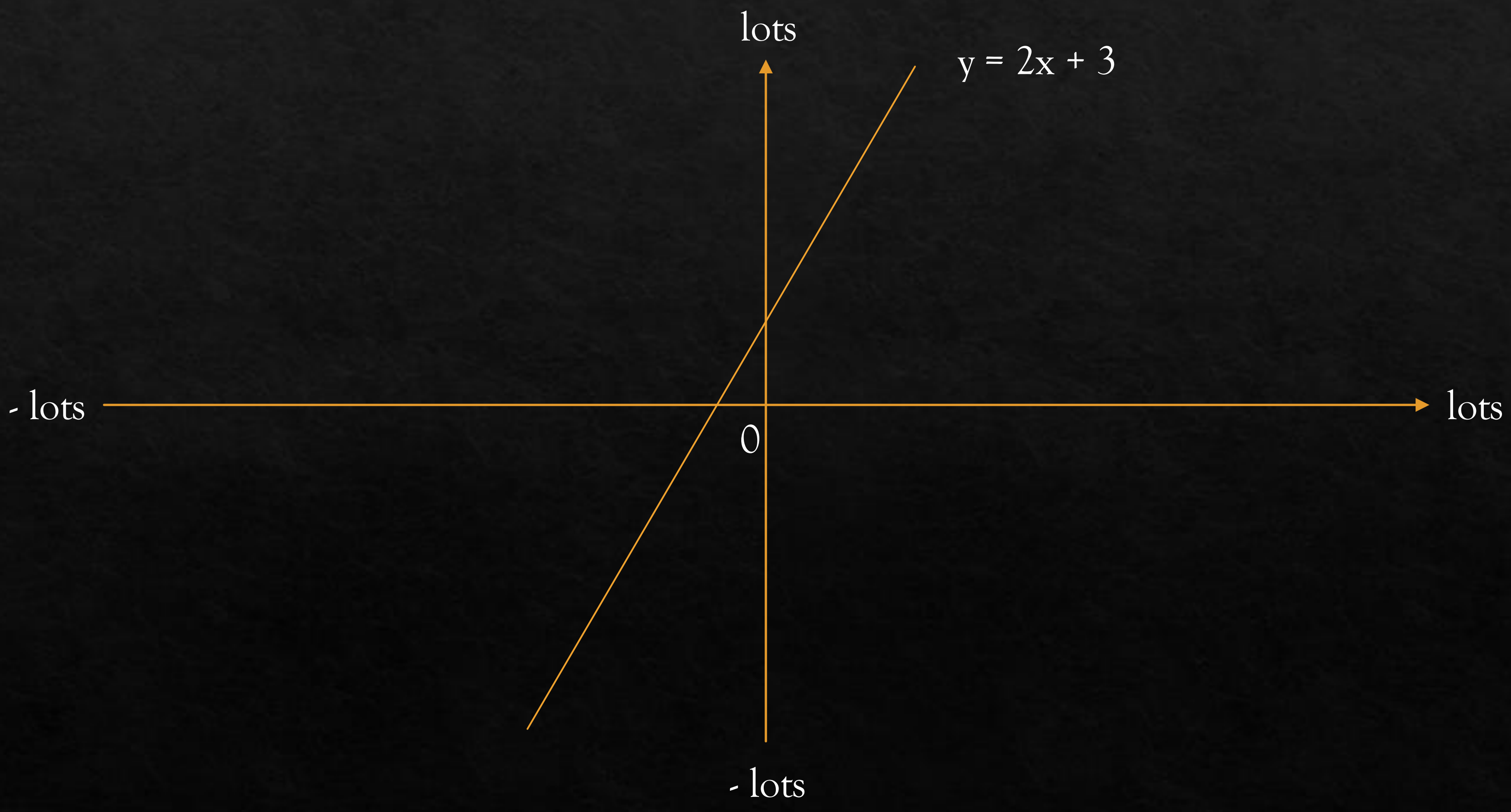
Applications in geometry



@hatcat01

Applications in geometry

- $a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$

@hatcat01

Applications in geometry

- $a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$
- $a_1x_1 + a_2x_2 = b$

Applications in geometry

- $a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$
- $a_1x_1 + a_2x_2 = b$
- $ax + by = c$

Applications in geometry

- $a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$
- $a_1x_1 + a_2x_2 = b$
- $ax + by = c$
- $by = -ax + c$

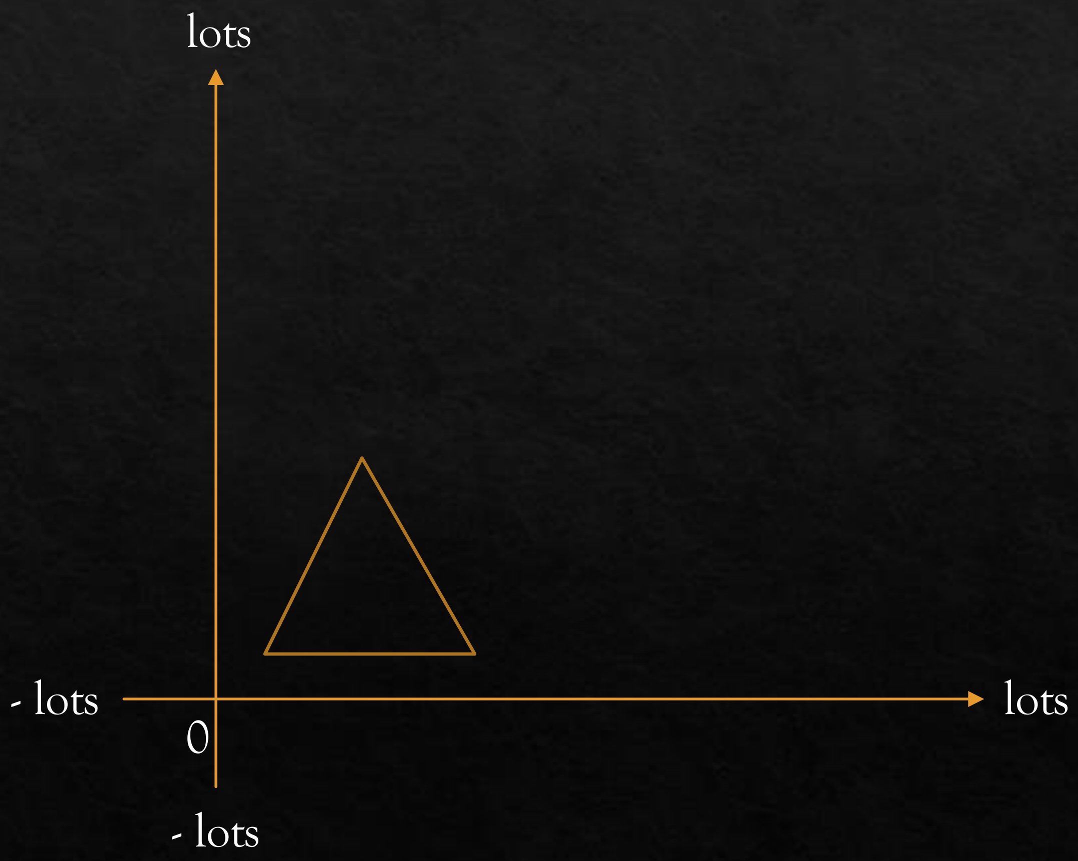
Applications in geometry

- $a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$
- $a_1x_1 + a_2x_2 = b$
- $ax + by = c$
- $by = -ax + c$
- $y = mx + c$

@hatcat01

Applications in geometry

- (x, y)
- Translate
- $(x, y) + (a, b) = (x + a, y + b)$



@hatcat01

Applications in geometry

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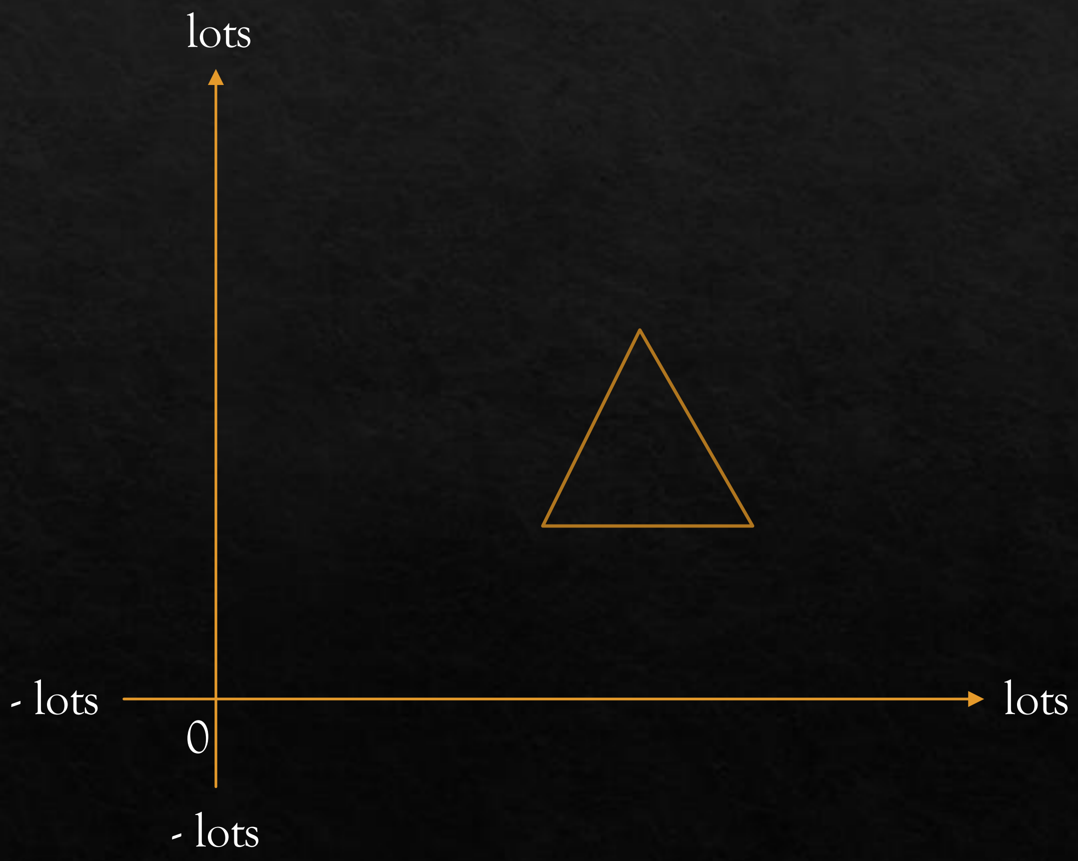
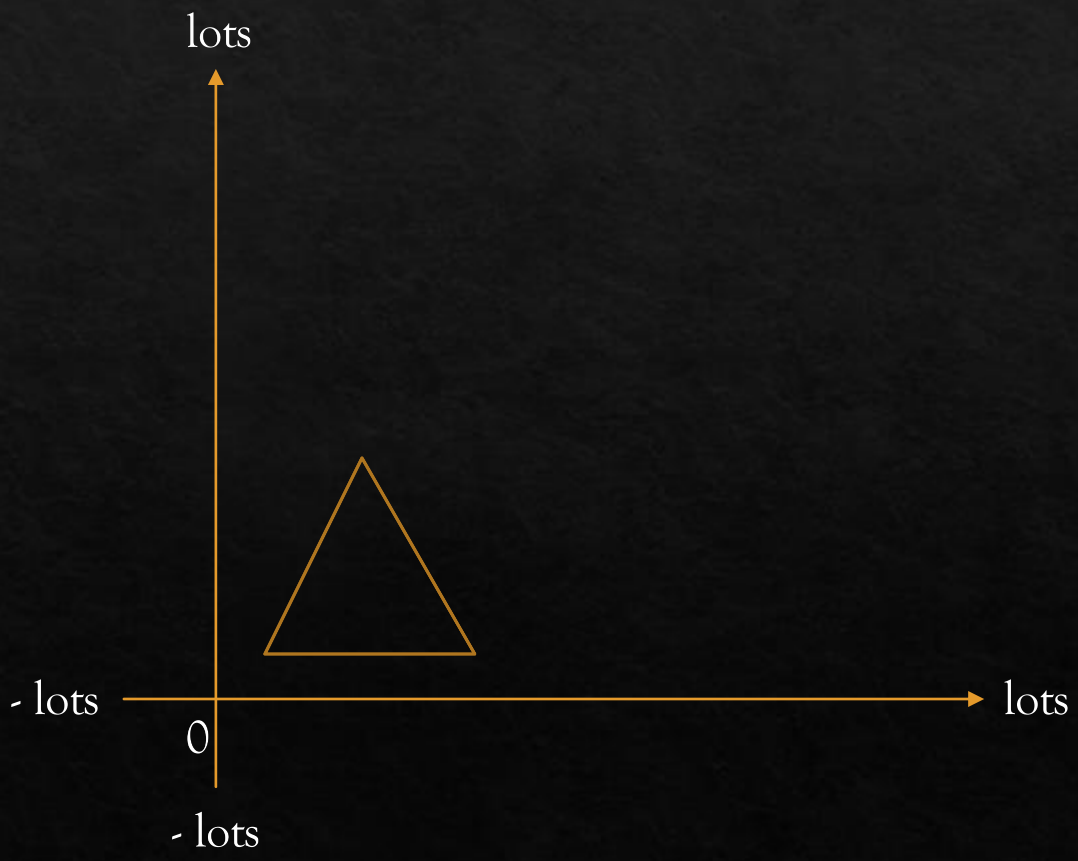


Applications in geometry

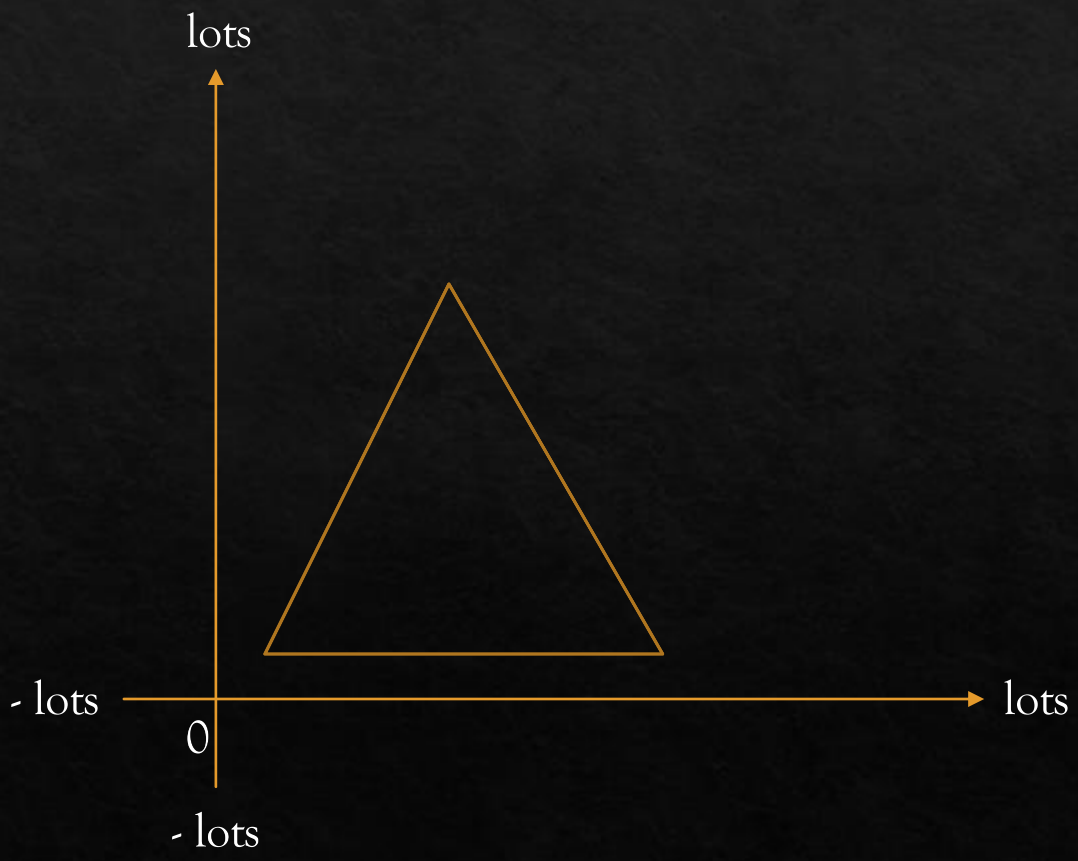
- (x, y)
- Scale
- (x, y) * 2 = (2x, 2y)
- (x, y) * (2 0) = (2x, 2y)
  (0 2)



@hatcat01

Applications in geometry

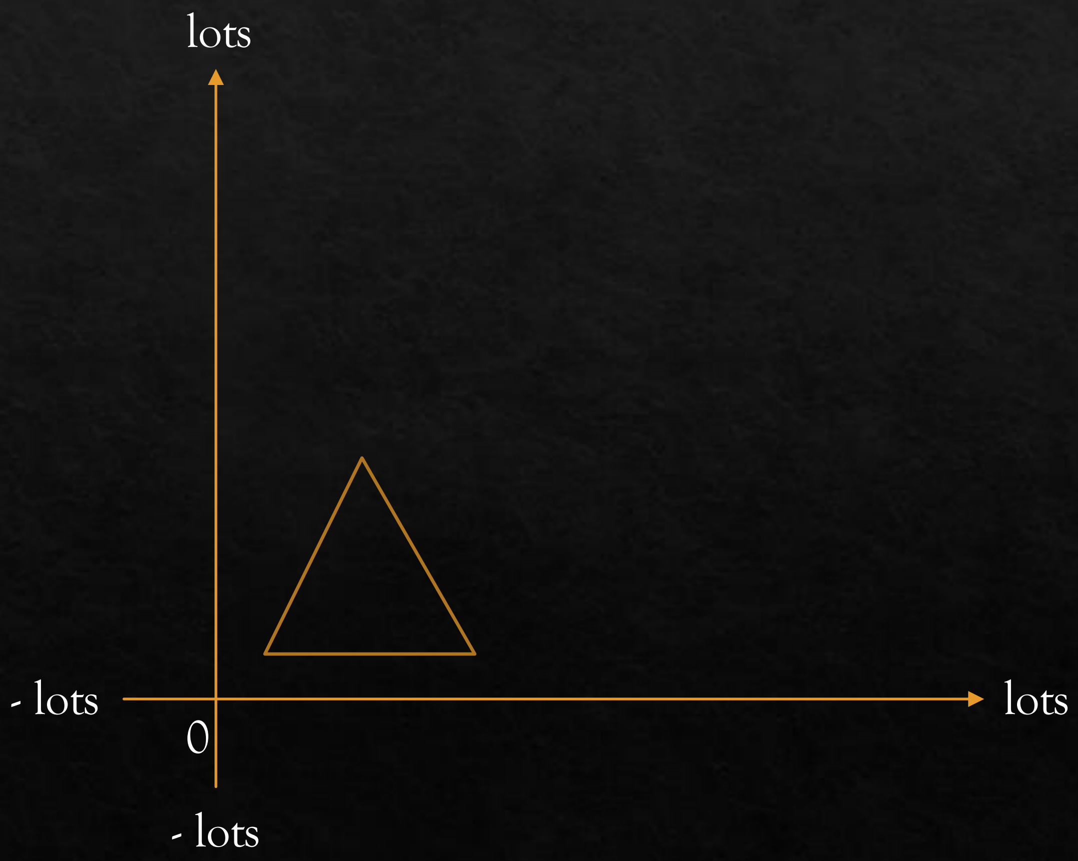
- (x, y)
- Scale
- (x, y) * 2 = (2x, 2y)
- $(x, y) * \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = (2x, 2y)$



@hatcat01

Applications in geometry

- (x, y)
- Shear
- $(x, y) * \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = (x, 2x + y)$



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Applications in geometry

- (x, y)
- Shear
- (x, y) * (1 2) = (x, 2x + y)
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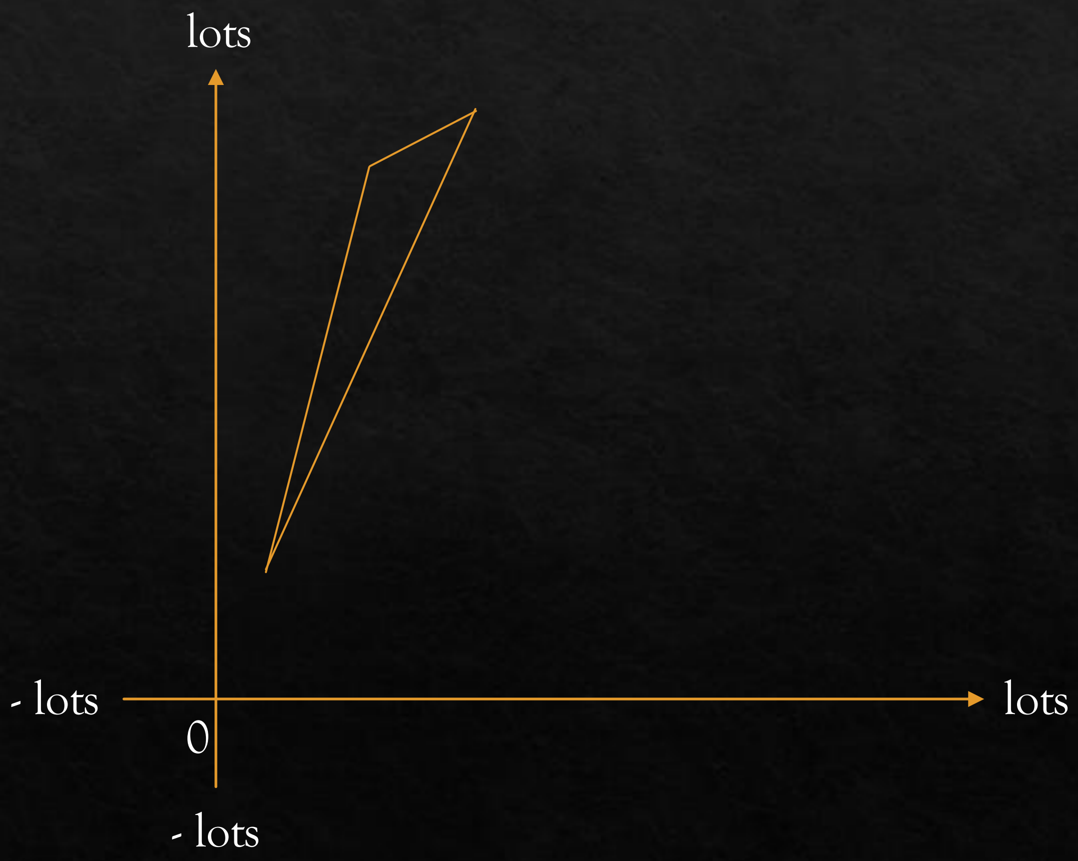

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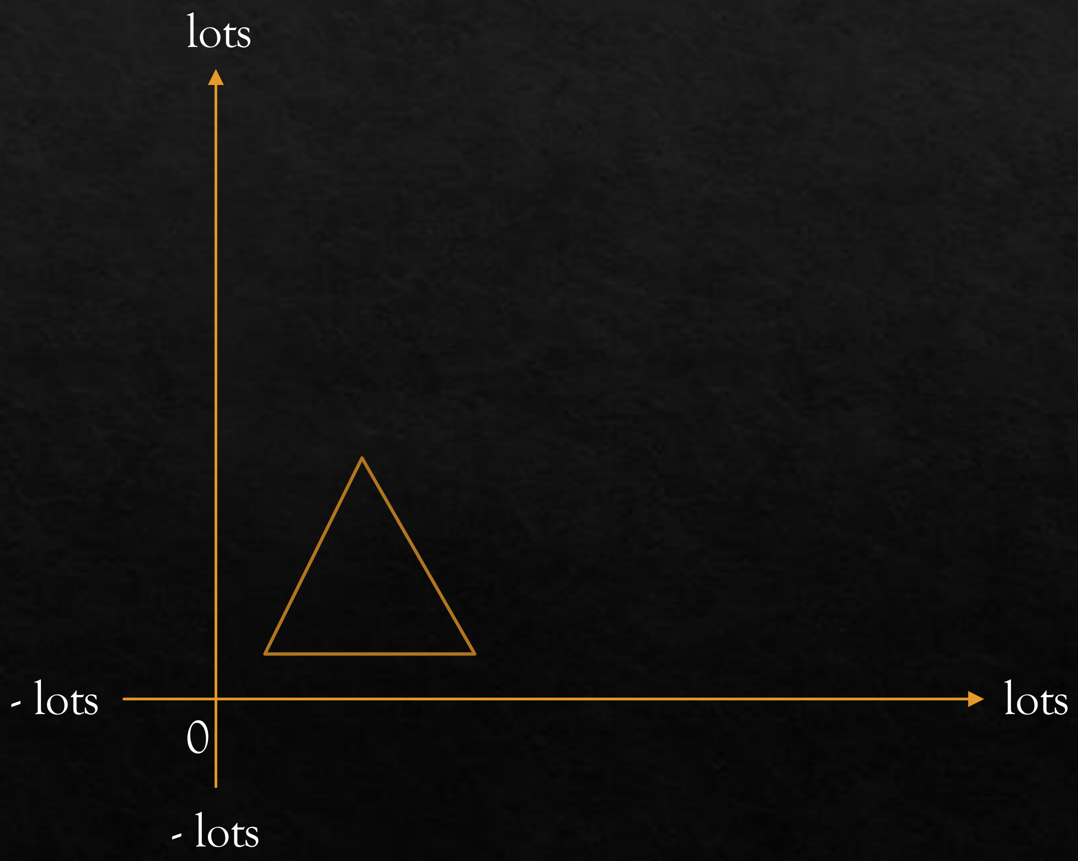
Applications in geometry

- (x, y)
- Reflect
- (x, y) * (1 0) = (x, -y)
  (0 -1)



@hatcat01

Applications in geometry

- (x, y)
- Reflect
- (x, y) * (1 0) = (x, -y)
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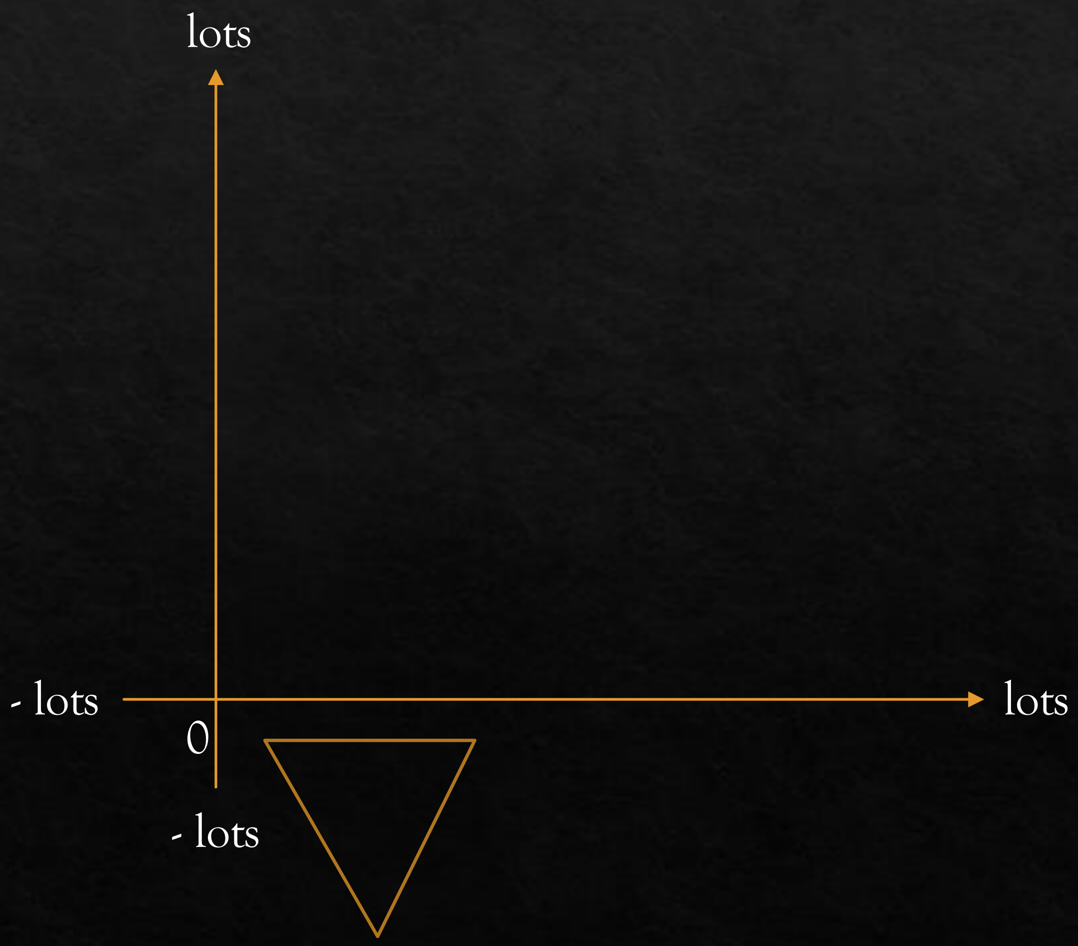
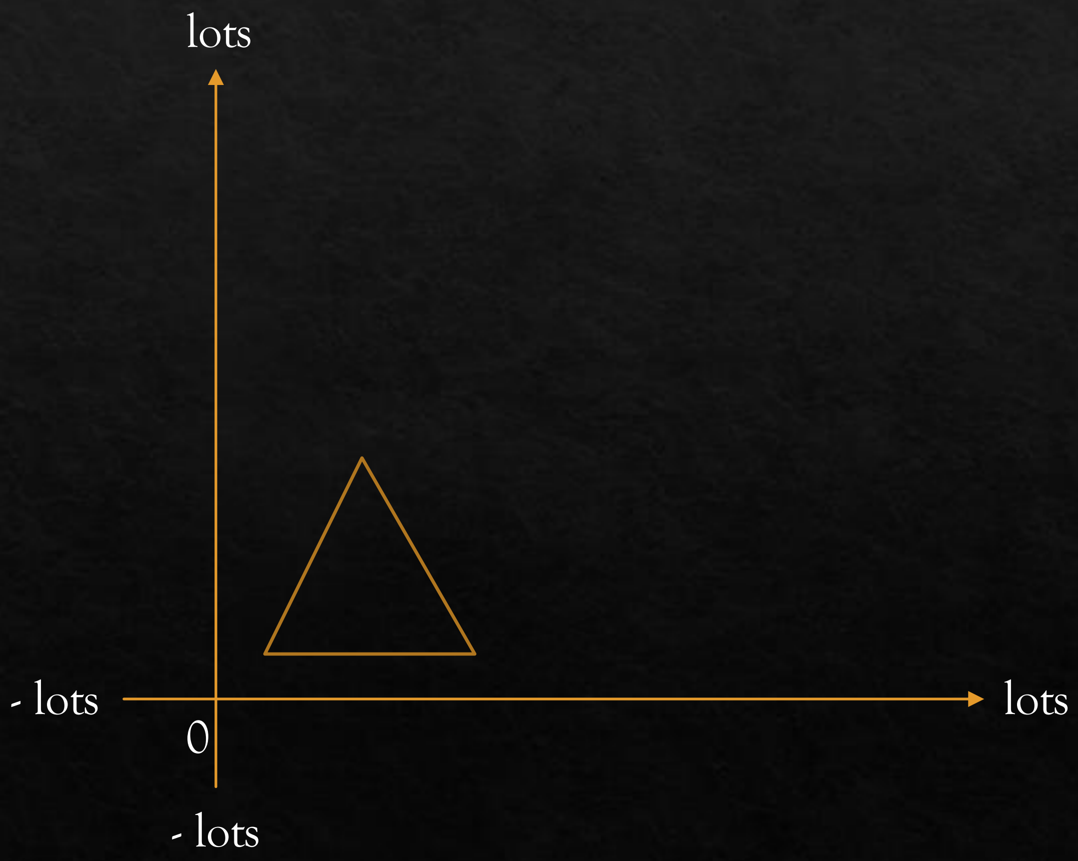


@hatcat01

Applications in geometry

- (x, y)
- Rotate
- (x, y) * (cos a –sin a)
  (sin a   cos a)
  = (x * cos a + y * sin a,
  -x * sin a + y * cos a)



@hatcat01

Applications in geometry

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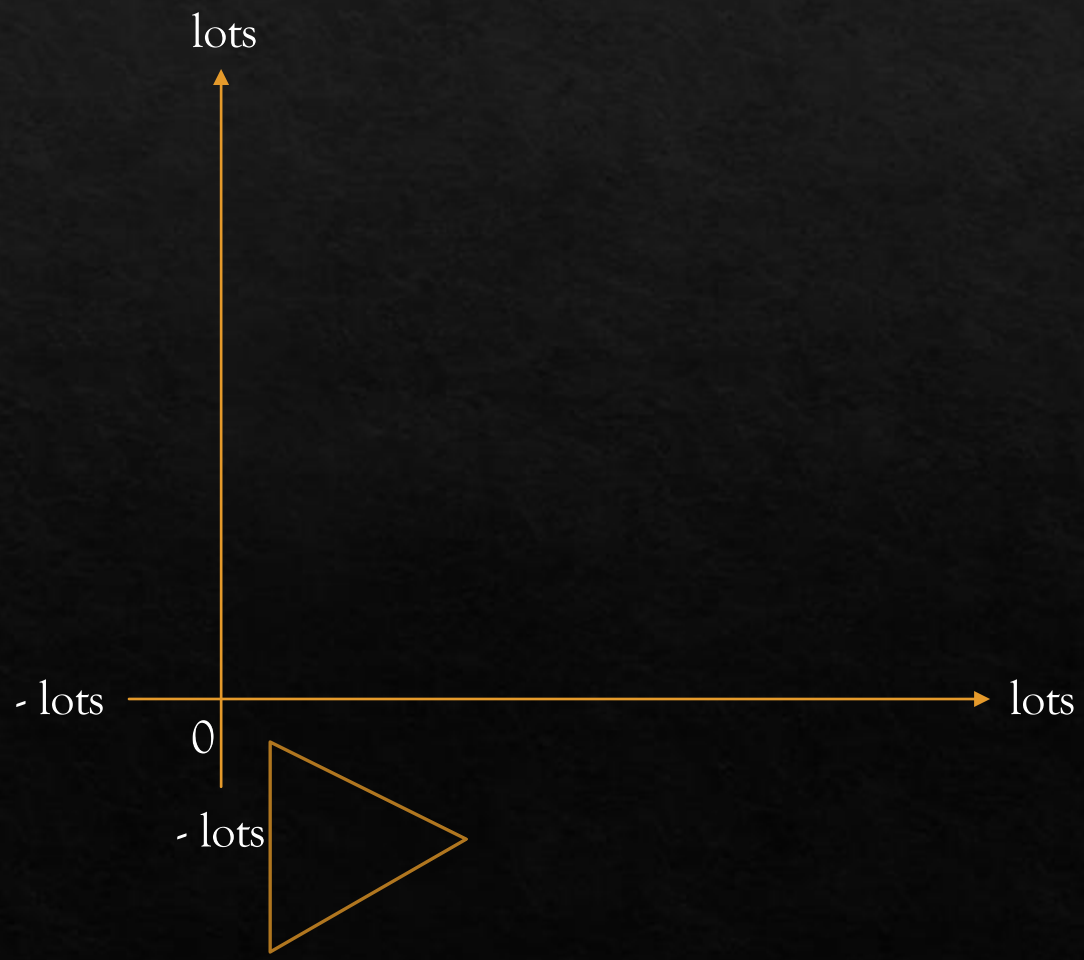

@hatcat01

Applications in geometry

- Boost.Geometry

Applications in geometry

- Boost.Geometry
- Barend Gehrels

Applications in geometry

- Boost.Geometry
- Barend Gehrels
- Geometry classes

Applications in geometry

- Boost.Geometry
- Barend Gehrels
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- Dimension agnostic

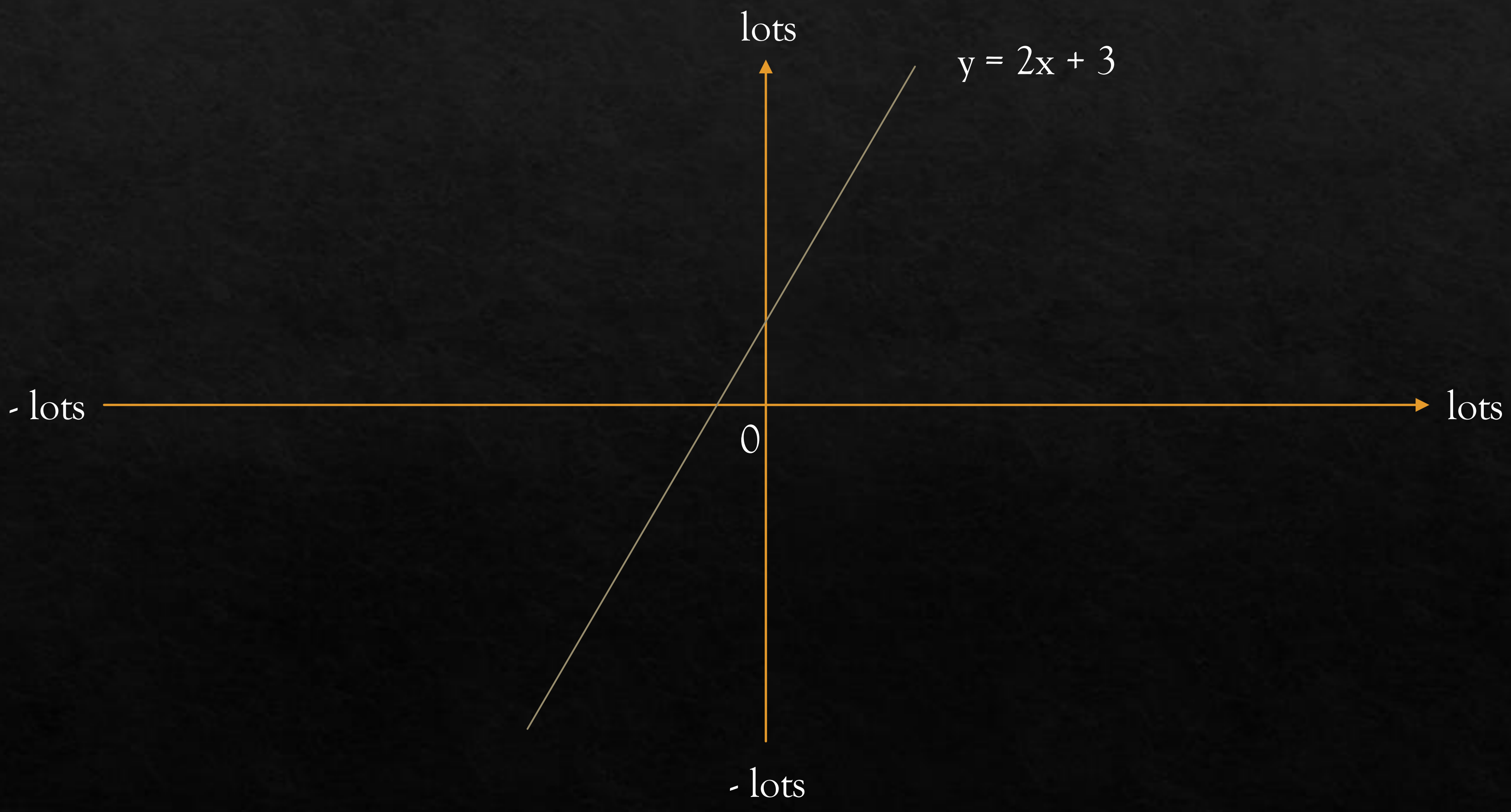
Applications in geometry

- Boost.Geometry
- Barend Gehrels
- Geometry classes
- Dimension agnostic
- Distance

Applications in geometry

- Boost.Geometry
- Barend Gehrels
- Geometry classes
- Dimension agnostic
- Distance
- Coordinate-system agnostic

Applications in geometry



@hatcat01

Applications in geometry

- ```
  struct line
  {
    float gradient;
    float y_intercept;
  };
  ```
  ```

Applications in geometry

```
struct line
{
  float gradient;
  float y_intercept;
};

struct line_segment
{
  point p1;
  point p2;
};
```

Applications in geometry

- Q

Applications in geometry

- Q
- 3244.7482

@hatcat01

Applications in geometry

- ```
  struct line
  {
    std::vector<point> points;
  };
  ```
  ```

Applications in geometry

- ```
  struct line
  {
    float gradient;
    float y_intercept;
  };
  ```

@hatcat01
  ```

Applications in geometry

- struct line

```
{
  float gradient;
  float y_intercept;
  point p1;
  point p2;
};
```

Applications in geometry

- ```
  struct line
  {
    float gradient;
    float y_intercept;
    point p_begin;
    point p_end;
  };
  ```

@hatcat01
  ```

Applications in geometry

- ```
  struct bezier
  {
    point p_begin;
    point p_control;
    point p_end;
  };
  ```
  ```

Applications in geometry

- $y = x - 1$
  $y = 2x - 4$

@hatcat01

Applications in geometry

- $y = x - 1$
  $y = 2x - 4$
- $0 = x - 3$
  $x = 3$

Applications in geometry

- $y = x^2$
  $y = x + 3.9$

Applications in geometry

- $y = x^2$
  $y = x + 3.9$
- $0 = x^2 - x - 3.9$
  $x = 0.5 \pm \sqrt{(16.6)}/2$

@hatcat01

Applications in geometry

- ```
  y = x - 2.3
  y = x/3
  ```
  ```

Applications in geometry

- y = x - 2.3
  y = x/3
- 0 = 2x/3 - 2.3
  x = 3.45

Applications in geometry

- `bool intersects(line a, line b);`

Applications in geometry

- `bool intersects(line a, line b);`
- `FLT_MIN` vs `FLT_EPSILON`

Applications in geometry

- `bool intersects(line a, line b);`
- `FLT_MIN vs FLT_EPSILON`
- `bool intersects(line a, line b, float epsilon);`

What to expect

What is linear algebra?

What is a linear algebra library?

Customising the library

Applications in colour

Applications in geometry

https://wg21.link/p1385

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ACCU 2021
VIRTUAL EVENT

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Engineering

undo

mosaic
CONSULTANTS TO FINANCIAL SERVICES

C++ and Linear Algebra

Guy Davidson